Biol 301 Spring 2001
Assignment 3
Due Monday, January 29, 2001

Reading assignment: Handout from Yodzis, Introduction to Theoretical Biology.

1. Solve Problem 4 on page 39 in Hastings, Population Biology.

2. Read the section about iteroparity versus semelparity on pages 27-28 in Hastings, Population Biology. Why is there no approach to the stable age distribution in this case? Argue both mathematically and biologically.

3. In a population of an imaginary organism that dies at the end of the third year of its life the average number of offspring of 0-year-olds in their first year of life is 1/3, the average number of offspring of 1-year-olds in their second year is 4, and the average number of offspring of 2-year-olds in their last year is 2. The probability that 0-year-old survives its first year to become a 1-year-old is 2/3, and the probability that a 1-year-old survives the second year to become a 2-year-old is 1/2. Death is certain after 3 years. Set up a Leslie matrix model for this population. How would you go about determining the long-term growth rate and the stable age distribution for this population? (Just explain; no calculations needed.)

4. Consider a population in which the average number of offspring per individual varies with population, e.g. due to competition for resources. Specifically, assume that if x is the population size in a given year, then the average number of offspring per individual is given by the function

$$f(x) = \lambda \cdot \exp\left(1 - x/K\right),$$

   where λ and K are (demographic) parameters.

   (a) Plot the function $f(x)$ for various values of λ and K (either by hand or using Mathematica).

   (b) Think of possible biological interpretations of the parameters λ and K.

   (c) Formulate a population dynamical model for the population size in the next year, $x(t+1)$, as a function of the population size in the present year, $x(t)$.

5. Consider the linear population dynamical model

$$x(t+1) = \lambda \cdot x(t),$$

   where $x(t)$ and $x(t+1)$ are population sizes in subsequent year, and λ is the average number of offspring per individual, which is assumed to be constant, i.e. independent of population size. Use the graphical method of cobwebbing to illustrate the dynamics of this population in the two cases when $\lambda > 1$ and when $\lambda < 1$. What happens when $\lambda = 1$?

6. Consider the non-linear population model

$$x(t+1) = \frac{\lambda \cdot x(t)}{1 + x(t)},$$

where $x(t)$ and $x(t+1)$ are population sizes in subsequent year, and λ is the average number of offspring per individual under ideal circumstances, i.e. if there is no competition for resources.

(a) Plot $x(t+1)$ a function of $x(t)$ for various values of λ (consider both $\lambda < 1$ and $\lambda > 1$).

(b) Use the method of cobwebbing to illustrate the dynamics of this population in the various cases considered in (a).

7. Consider the same population as in problem 7., but now assume that in addition to competition for resources there is also predation, which occurs in such a way that after reproduction has taken place, a constant number of individuals h are removed from the population in each year. Therefore, the population dynamical model changes to

$$x(t+1) = \frac{\lambda \cdot x(t)}{1 + x(t)} - h.$$

(a) Use the method of cobwebbing to determine the dynamics of this population for various values of λ and h.

(b) How do the dynamics change compared to the population in problem 7.? How does the equilibrium population size change?

(c) What is wrong with this model?

(d) Can you suggest a better way to incorporate constant predation?