**Biol 301 Spring 2001**
**Assignment 1: Exponential Growth (and decay)**
Due Wednesday, January 10, 2001

**Reading assignment:** Hastings, Population Biology, pages12-24.

1. Some math review first:

   **(a)** Solve $\ln(a^t) = b$ for $t$.

   **(b)** Solve $a^t = b$ for $t$

   **(c)** Solve $\ln(ax)+\ln(bx) - \ln(c) = d$ for $x$.

   **(d)** Find the derivative $\frac{df}{dx}$ for the following functions: $f(x) = \ln(x)$, $f(x) = \exp(ax)$, $f(x) = \ln(x^a)$.

   **(e)** Find the following integrals: $\int \exp(ax)dx$, $\int \frac{dx}{x}$, $\int \ln(x)dx$ (use integration by parts for the last one).

2. Consider a population of yeast cells growing on an agar plate. Suppose that each cell divides into two daughter cells once every hour.

   **(a)** Suppose we start out with a single yeast cell. Write down the number of yeast cells after 1 hour, 2 hours, 3 hours, 5 hours, 10 hours.

   **(b)** Let $t$ be time in hours, and let $N(t)$ be the size of the yeast population after $t$ hours. Formulate an equation that describes the dynamics of the yeast population by giving $N(t + 1)$, the population size one hour later, as a function of $N(t)$.

   **(c)** Suppose that there is enough food on the plate to support a population of $10^8$ yeast cells. Starting with one cell, how many hours does it take until the yeast population reaches this size?

3. Consider the following statement in Michael Crichton's book *Andromeda Strain* (Dell, N.Y., 1969, p. 247):

   "The mathematics of uncontrolled growth are frightening. A single cell of the bacterium E. coli would, under ideal circumstances, divide every twenty minutes. That is not particularly disturbing until you think about it, but the fact is that bacteria multiply geometrically: one becomes two, two become four, four become eight, and so on. In this way it can be shown that in a single day, one cell of E. coli could produce a super-colony equal in size and weight to the entire planet Earth."

   Assume that Crichton's ideal circumstances hold and determine whether his statement is correct under the realistic assumption that the mass of an E. coli bacterium is roughly $10^{-12}$ grams and by taking into account that the mass of the earth is roughly $5.9763 \cdot 10^{24}$ kilograms. (*Hint*: Use the methods of the previous problem to calculate the number of bacteria present after one day.)

4. Going back to the population of yeast cells in problem 2, assume that after the population has reached its limiting size of $10^8$, individuals stop dividing and instead start to die off due to lack of food. Assume that for any individual cell that is alive the chance of dying during the next two hours is $1/3$.

   **(a)** Formulate a model for the dynamics of the yeast population under this assumption. (*Hint*: Choose 2 hours as the basic unit of time, and describe $N(t+1)$ as a function of $N(t)$.)

   **(b)** How long does it take for the population to die out? (*Hint*: how many time units does it take for $N(t)$ to be smaller than 1?)

   **(c)** Can you think of reasons for why your answer in b) might be wrong? (*Hint*: Keep in mind that the fate of individuals is given by the expected probability of surviving a certain time period. In reality, some might live longer and some might live shorter than expected. As long as the population size is large these differences will average out, but what happens when population sizes get very small?)

5. Imagine a population with two age classes, so that each individual in the population is either a 'juvenile' or an 'adult'. Suppose that only adults reproduce, and that they do so by producing on average 0.9 juveniles per year. Assume also that adults die after reproduction. Suppose further that in each year $1/2$ of all the juveniles survive to become adults, while the other half dies.

   **(a)** Formulate a model for the dynamics of this population with one year as the basic time unit. (Hint: you will end up with two equations, one describing the size of the juvenile population in the next year as a function of the adult population size in the present year, and one describing the adult population size next year as a function of the juvenile population size in the present year.)

   **(b)** Starting out with 100 juveniles and 200 adults, give the population sizes (juveniles, adults, and total) in the following 5 years.

   **(c)** What is the long term fate of the populations? What is the reason for this in terms of the demographic parameters, i.e. in terms of average reproductive output per adult individual and of juvenile survival probability? (Hint: no calculations needed.)

   **(d)** Suggest a way of 'salvaging' the population by changing the demographic parameters so that the population becomes viable (i.e. survives in the long run).