MATH 300 ASSIGNMENT 7: DUE MAR 11 (FRI) IN CLASS

(1) Using ratio or comparison test, show that the following series converge:
   a) $\sum_{j=3}^{\infty} \frac{2^j}{j!}$
   b) $\sum_{j=1}^{\infty} \frac{\sin(2j)}{3^j}$

(2) Consider the sequence of functions $F_n(x) = x^n$ on (0,1) which converges pointwise to $F(x) = 0$. Show that for $0 < x < 1$,
   $$|F_n(x) - F(x)| < \frac{1}{2}$$
   when and only when $n > \log 2 / \log(1/x)$. This problem shows that the sequence $F_n(x)$ does not converge uniformly on (0,1).

(3) Verify the following Taylor series by finding a general formula for $f^{(j)}(z_0)$:
   $$\cosh z = \sum_{j=0}^{\infty} \frac{z^{2j}}{(2j)!}, \quad z_0 = 0.$$  

(4) By using the Cauchy product, find first three nonzero terms in the MacLaurin expansion of the function $e^z \sin z$.

(5) Find and state the convergence properties of the Taylor series of the function $f(z) = z/(1-z)$ around the point $z_0 = i$. 