MATH 300 ASSIGNMENT 1: DUE JAN 15 (FRI) IN CLASS

(1) Let \( z = 3 - 4i \). Plot the points \( z, -z, \bar{z} \) and \( 1/z \) in the complex plane.

(2) Describe the set of points \( z \) in the complex plane that satisfies
\[
\left| (1 + i)z - 2 \right| = 4.
\]
Plot it in the complex plane.

(3) Prove that if \( |z| = 1 \) and \( z^4 \neq 1 \), then
\[
\text{Re}\left( \frac{1}{1 - z^4} \right) = \frac{1}{2}.
\]

(4) Find \( |z|, \text{Arg}(z) \) and \( \text{arg}(z) \) of the following complex numbers:
   a) \( z = 6 - 6i \)
   b) \( z = e^w \), where \( w = \sqrt{2}\pi e^{i\pi/4} \).

(5) Write each of the following numbers in the polar form \( re^{i\theta} \):
   a) \( \frac{1+i}{3+i} \)
   b) \( (\sqrt{3} - i)^7 \).

(6) Compute the following integrals (simplify your answer to the best possible, hint: use \( \cos \theta = (e^{i\theta} + e^{-i\theta})/2 \) and similar formula for \( \sin \theta \)):
   a) \( \int_0^\pi \cos^3 \theta d\theta \).
   b) \( \int_0^\pi \sin^6(2\theta) d\theta \).

(7) Find all values of the following
   a) \( (-81)^{\frac{1}{4}} \).
   b) \( \left( \frac{2i}{1-i} \right)^\frac{1}{5} \).

(8) Suppose \( u = u(x, y) \) is a real-valued function in a domain \( D \) and satisfies
\[
\frac{\partial u}{\partial x}(x, y) = y^2, \quad \text{and} \quad \frac{\partial u}{\partial y}(x, y) = 2xy,
\]
for any \( (x, y) \) in \( D \). Determine \( u(x, y) \) up to an additive constant.

(9) Let \( b \) and \( c \) be complex constants. Prove that the solutions of the equation
\[
z^2 + bz + c = 0
\]
are given by the usual quadratic formula
\[
z = \frac{-b \pm \sqrt{b^2 - 4c}}{2},
\]
where \( \sqrt{b^2 - 4c} \) denotes one of the values of \( (b^2 - 4c)^{\frac{1}{2}} \). Solve the equation
\[
z^2 - (2 + i)z + 3 + i = 0.
\]
Simplify your answer to the best possible form.