Primitive Prime Divisors in Arithmetic Dynamics

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Diophantine Geometry and Arithmetic Dynamics

Diophantine geometry: studies K-rational points on varieties defined over K where K is arithmetically interesting (e.g.: number fields, function fields,...)

Dynamics: studies a self-map $\phi : S \to S$, and all the iterates ϕ^n for $n \in \mathbb{N}$.

Arithmetic dynamics: when K is arithmetically interesting, S is a variety over K, and ϕ is a K-morphism.

Example: a special case of a joint result with Chad Gratton and Thomas Tucker (to appear Bulletin London Math. Soc.):

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Theorem

Let $\phi(X) \in \mathbb{Q}[X]$ of degree $d \geq 2$. Let $a \in \mathbb{Q}$ having infinite ϕ-orbit. Assume the ABC conjecture.

(a) Assume that $\phi(X)$ does not have the form uX^d. Then for all $n >> 0$, there is a prime p (depending on n) such that $v_p(\phi^n(a)) > 0$ and $v_p(\phi^m(a)) \leq 0$ for all $1 \leq m < n$.

(b) Assume that $\phi^n(X)$ has a square-free factor in $\bar{\mathbb{Q}}[X]$ for every n. Then for all $n >> 0$, there is a prime p (depending on n) such that $v_p(\phi^n(a)) = 1$ and $v_p(\phi^m(a)) \leq 0$ for all $1 \leq m < n$.

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