University of British Columbia
Math 200, Section 104
Quiz 2; Friday, 30 September 2016
Student Name:
SID:

Time = 10 minutes. Notes and calculators are NOT allowed. This quiz is worth 6 points. You need to justify your answers; giving only the final answers without any explanation will give you no credit.
Make sure you have clearly written your name and SID before attempting these questions.

**Question.** Let $L_1$ and $L_2$ be the lines with the symmetric equations:

$L_1: \quad \frac{x - 1}{2} = \frac{y - 1}{2} = \frac{z + 1}{3}$

$L_2: \quad \frac{x - 3}{2} = \frac{y - 2}{-1} = \frac{z - 2}{-2}$

(a) Write down parametric equations for $L_1$ and $L_2$. (2pts)
(b) Find an equation for the plane $S$ that contains $L_1$ and is parallel to the $x$-axis (hint: compare the normal vector of $S$ with $L_1$ and the $x$-axis). (2pts)
(c) Using the parametric equations you found in (a), check if $L_1$ and $L_2$ intersect. If they intersect, find the point of intersection. (2pts)

Some formulas:
(1) $\text{proj}_\vec{u} \vec{v} = \frac{\vec{u} \cdot \vec{v}}{\vec{v} \cdot \vec{v}} \vec{v}$

(2) Distance from point $Q$ to line $L$: pick $P$ on $L$, take $\frac{||\overrightarrow{PQ} \times \vec{d}||}{||\vec{d}||}$.  

(3) Distance between skew lines $L_1$ and $L_2$: pick $P_1$ on $L_1$, $P_2$ on $L_2$, let $\vec{c} = \overrightarrow{P_1P_2} \cdot \vec{d}$, take $\frac{|\overrightarrow{P_1P_2} \cdot \vec{d}|}{||\vec{d}||}$.

(4) Plane $S$ having equation $ax + by + cz + d = 0$, pick $P$ on $S$. Distance from point $Q(x_1,y_1,z_1)$ to $S$ is:

$$\frac{|\overrightarrow{PQ} \cdot \vec{n}|}{||\vec{n}||} = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}.$$  

**Answer:**

(a) $L_1$: \[
\begin{align*}
x &= t \\
y &= 1 + 2t \\
z &= -1 + 3t
\end{align*}
\]

(b) $L_2$: \[
\begin{align*}
x &= 3 + 2t \\
y &= 2 - t \\
z &= -2t
\end{align*}
\]

(b) $\vec{n}$ is orthogonal to $\vec{d}_1 = \langle 1, 2, 3 \rangle$ since $\vec{d}_1 \parallel L_1$

and $\vec{n}$ is orthogonal to $\vec{i} = \langle 1, 0, 0 \rangle$ since $\vec{i} \parallel x$-axis

$$\vec{n} = \begin{vmatrix} i & j & k \\ 1 & 2 & 3 \\ 1 & 0 & 0 \end{vmatrix} = \langle 0, 3, -2 \rangle$$
Pick the point \((0, 1, -1)\) on \(L_1\) hence on the plane.

Answer:

\[3(y-1) - 2(z+1) = 0\]

(c) Check if there's a solution to

\[
\begin{aligned}
    t &= 3 + 2s \\
    1 + 2t &= 2 - s \\
    -1 + 3t &= -2s
\end{aligned}
\]

(\text{review the notes about "lines" to see why we have this})

Consider first 2 equations:

\[
\begin{aligned}
    t &= 3 + 2s \\
    1 + 2t &= 2 - s 
\end{aligned}
\]

\[\Rightarrow 1 + 2(3 + 2s) = 2 - s \]

\[5 = -5s \]

\[s = -1, \quad t = 1\]

Plug \((t=1, s=-1)\) to the 3rd eq: \(2 = 2\) \(\checkmark\)

Two lines intersect at the point (plug \(t=1\) in eq for \(L_1\)) \((1, 3, 2)\)