Eq: \[ f(x,y) = 2x + 3y + 4 \]

a) Domain?

b) Graph of \( f(x,y) \) (without drawing)?

---

* Level curves of \( f(x,y) \):

All these mean the same thing:

- the trace/cross section in \( z = k \) of the graph of \( f(x,y) \)
- the level curve/contour/contour line of \( f(x,y) \) at level \( k \)
- the curve in \( \mathbb{R}^2 \) with equation \( f(x,y) = k \)
Eq 1: Show figure 12.4 in p. 678 APEX (level $c = 0, 0.2, 0.4, 0.6, 0.8, 1$)

Note: here the level curve at level $c = 1$ is just a point.

Eq 2: Figure 12.5 in p. 679. If you are not given $f(x, y)$, can you tell $f(0, 2) = ?$

Answer:

How to find level curves (algebraically)? To find level curve at level $k$:

... 

Eq: find level curves of

a) $f(x, y) = x^2 + y^2$

b) $f(x, y) = \sqrt{x^2 + y^2}$
c) \( f(x, y) = \sin(x-y) \) (to check our answer: type "level curve \( f(x, y) = \sin(x-y) \)" on WolframAlpha)

Spacing of level curves: so from previous eq, \( f_1(x, y) = x^2 + y^2 \) and \( f_2(x, y) = \sqrt{x^2 + y^2} \) have the same level curves?

\( \Rightarrow \) Not really, both yes & no

Why yes?

Why no? Evenly spaced vs not evenly spaced
Type "level curve $x^2 + y^2$" and "level curve $\sqrt{x^2 + y^2}$" and see the difference.

* 3 or more variables, level surfaces

* For function in 3 variables
  - Domain $D = \text{subset of } \mathbb{R}^3$
  - Each $(x, y, z)$ in $D$ gives $w = f(x, y, z)$
  - Graph $w = f(x, y, z)$: "solid object" in the 4-d space $\mathbb{R}^4$; there’s one extra dimension given by the $w$-axis

  - level surface: function $f(x, y, z)$ level $k$
    
    $\Rightarrow$ surface $f(x, y, z) = k$ in $\mathbb{R}^3$

An extra remark (read at home. This is not important for now; but later on you will have 2 formulas for tangent planes & I hope this can help avoid confusion)

A surface in $\mathbb{R}^3$ is usually given in one of the following 2 forms:

Form 1: graph of a function $f(x, y)$

Equation of your surface is $z = f(x, y)$

eg: graph of $\sqrt{9 - x^2 - y^2}$ is an upper hemisphere as discussed in previous lecture.
Form 2: level surface of a function \( F(x,y,z) \)

Eq of your surface is \( F(x,y,z) = k \)

eg: the sphere of radius 3 centered at \((0,0,0)\) is given by \( x^2 + y^2 + z^2 = 9 \)

Quick remark:

* Form 2 is more general: the graph \( z = f(x,y) \) can also be described as the level surface \( F(x,y,z) = 0 \) with, obviously, \( F(x,y,z) = f(x,y) - z \)

* Although you can describe, say, the upper hemisphere as graph of \( \sqrt{9-x^2-y^2} \) (or lower hemisphere as graph of \( -\sqrt{9-x^2-y^2} \)), you can't describe the whole sphere as graph of a function \( f(x,y) \)

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§ 12.2 Limits and Continuity

* Very briefly just to provide some intuition

* Ignore all the rigorous definitions in APEX.

Review/Motivation from 1-var functions:

* \( \lim_{x \to 0} \frac{1}{x} \quad \text{< exists} \quad \text{?} \)

* \( \lim_{x \to 0} \frac{1}{x} \quad \text{< does not exist (DNE)} \)
Although 0 does not belong to the domain of \( \frac{\sin x}{x} \), the limit \( \lim_{x \to 0} \frac{\sin x}{x} \) exists and is equal to DNE.

Define \( f(x) : \mathbb{R} \to \mathbb{R} \)

\[
f(x) = \begin{cases} \frac{\sin x}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}
\]

Then \( f \) is continuous everywhere since continuous at \( x \neq 0 \) since continuous at \( x = 0 \) since

2-variable functions:

\[
\lim_{(x,y) \to (x_0,y_0)} f(x,y) = L \quad \text{means: if } (x,y) \text{ approaches } (x_0,y_0) \text{ in an arbitrary manner then } f(x,y) \text{ approaches } L
\]

Eg 1:

\[
\lim_{(x,y) \to (0,0)} \frac{\sin (x^2 + y^2)}{x^2 + y^2} = \]
Eq. 2: \( \lim_{(x,y) \to (0,0)} \frac{x^2}{x^2 + y^2} \) exist?

a) let \((x,y)\) approach \((0,0)\) while \((x,y)\) on the diagonal line \(x = y\), e.g. take \(\left(\frac{1}{2}, \frac{1}{2}\right), \left(\frac{1}{3}, \frac{1}{3}\right), \left(\frac{1}{4}, \frac{1}{4}\right), \ldots\)

\[\text{then } \frac{x^2}{x^2 + y^2} \text{ approaches ?}\]

b) Should the number in (a) be the limit?

If not, what is wrong?

c) Explain why limit does not exist.
Continuity: say \( f(x, y) \) is continuous at \((x_0, y_0)\) if
- \((x_0, y_0)\) is in the domain,
- \( \lim_{(x,y) \to (x_0,y_0)} f(x,y) \) exists,
- and the limit is \( f(x_0, y_0) \).

All elementary functions (polynomial, rational, exp, log, trig) are continuous in their domain.

§12.3 Partial Derivatives (§14.3 in Stewart)
- I will start to refer to Stewart a lot (so it's good to have a hard copy or a pdf).
- Warm-up: \( a) f(x, y) = xe^y \), regard \( y \) as a constant, find derivative with respect to \( x \).

b) \( g(x, y) = xe^{xy} \), regard \( x \) as constant, find derivative \( \text{wrt} \ y \).