Eg 8: sketch a plane in $\mathbb{R}^3$

a) Need how many non-collinear points to determine a plane?

ans:

b) Sketch the plane rep. by $x + y + z = 1$

ans:

Eg 9 (a bit harder than previous ones)

a) Given 2 points $A$ and $B$ in $\mathbb{R}^3$, can you guess the set of points $P$ which are equidistant to $A$ and $B$?

b) Let $A(1, -2, 3), B(0, 1, 1)$, write down an equation representing the set of points $P(x, y, z)$ that are equidistant to $A$ and $B$.

(hint: set $||PA|| = ||PB||$, square both sides, then simplify)

Answer:
§ 10.2 Vectors

* Basic definition:
- Vector: a line segment with a direction

\[ \vec{PQ} \]

P: initial point  \quad Q: terminal point

length of the vector = length of the \( \| \overline{PQ} \| \) segment

- 2 vectors are equal if \(<\) same & \(<\) same
(need not coincide)

\[ \vec{u} = \vec{v} \quad \vec{u} \neq \vec{v} \]

* Vector addition and scalar multiplication: geometric motivation

Add 2 vectors:

\[ \overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC} \]
Q: How about $\vec{u} + \vec{v}$ where

answer: "move $\vec{v}$"

1. $\vec{u} \rightarrow \vec{v}$

or

2. Parallelogram law:

Multiply a (scalar) → real #

$c$: real #, $\vec{v}$: vector

$\vec{c} \vec{v}$:

- same direction as $\vec{v}$ if $c > 0$
- opposite direction to $\vec{v}$ if $c < 0$
- the zero vector if $c = 0$

length of $\vec{c} \vec{v}$

In our notation:

* Vector addition & scalar mult.: using coordinates for easy calculation (very important) called "components" in the book

In $\mathbb{R}^2$: given $\vec{u}$, move $\vec{v}$ so that origin = initial = origin
call terminal = $P$

$\Rightarrow$

represent $\vec{v}$
Eg 1: \( \vec{v} \) with initial \((1, 2)\) terminal \((4, 3)\)

What is the coordinate representation (also called component form) of \( \vec{v} \)?

Answer:

\[ <3, 1> \]

Notation \(<, >\) does not mean the point, it means the vector.

Formula: \( A(x_1, y_1), B(x_2, y_2) \) then \( \overrightarrow{AB} \) has component form

\[ \overrightarrow{AB} = \]
Addition, scalar mult., & length using component form:

\[ \overrightarrow{a} = \langle a_1, a_2 \rangle, \quad \overrightarrow{b} = \langle b_3, b_2 \rangle, \quad c: \text{ real number} \]

\[ \overrightarrow{a} + \overrightarrow{b} = \]

\[ c\overrightarrow{a} = \]

length of \( \overrightarrow{a} \) \[\| \overrightarrow{a} \| = \]

* Similar discussion & formulas for vectors in \( \mathbb{R}^3 \)

Eq: Determine if points lie on straight line (without drawing)

a) \( A(2, 4, 2), \ B(3, 7, -2), \ C(1, 3, 3) \)

Answer: (idea: check if \( \overrightarrow{AB} \) and \( \overrightarrow{AC} \) on the same line ("parallel" to each other))

\[ \rightarrow \text{check if } \overrightarrow{AB} = c \overrightarrow{AC} \text{ for some scalar } c \]

\[ \rightarrow \text{check if coordinates of } \overrightarrow{AB} \text{ & } \overrightarrow{AC} \text{ are proportional;} \]

Comment quotient = c
b) \(D(0,-5,5), \ E(1,-2,4), \text{ and } F(3,4,2)\)

\[\text{Eq: Find the midpoint of } AB \text{ in the previous eq}\]

\[\text{hint:}\]

\[\begin{align*}
\vec{OM} &= \square, \text{ the diagonal } \vec{OC} \\
&= \square \cdot (\vec{OA} + \vec{OB}) \\
\Rightarrow \text{ coordinate of } M &= \square (\text{coord. of } A + \text{coord. of } B)
\end{align*}\]

\[\text{Answer:}\]

\[\text{Properties of Vector Operations: see Theorem 84 in p.572 of the primary textbook}\]

\[\text{Roughly: all the natural properties that you expect}\]

\[\text{Unit vectors}\]

\[\text{Def: unit vector is a vector of }\]
* Given \( \vec{0} \) the zero vector \( (0,0) \) in \( \mathbb{R}^2 \), \( (0,0,0) \) in \( \mathbb{R}^3 \)

- The unit vector pointing
  - in the same direction of \( \vec{v} \) is \( \underline{\text{boxed}} \)
  - in the opposite direction of \( \vec{v} \) is \( \underline{\text{boxed}} \)

**EG:** Find the unit vector in the direction of \( \langle 1, 2, -3 \rangle \)

**Ans:**

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**Standard Unit Vectors**

In \( \mathbb{R}^3 \), let

- \( \vec{i} \) means
- \( \vec{j} \) means
- \( \vec{k} \) means

**EG:** \( 10\vec{k} + 100\vec{j} - 1000\vec{i} \)

\[ = \langle \frac{1000}{10}, \frac{-100}{10}, \frac{10}{10} \rangle \]

Conversely:

\( \langle 3, 4, 5 \rangle = \underline{\text{boxed}} \vec{i} + \underline{\text{boxed}} \vec{j} + \underline{\text{boxed}} \vec{k} \)

\( \langle a_1, a_2, a_3 \rangle = \underline{\text{boxed}} \vec{i} + \underline{\text{boxed}} \vec{j} + \underline{\text{boxed}} \vec{k} \)
Eq: (a bit tricky, do it yourself, not discussed in the lecture)

In $\mathbb{R}^2$, let $A(-1,3)$ and $B(5,2)$. Find a point $C$ such that the line through $O$ and $C$ bisect the angle $\angle AOB$. 