So \( E = \{ (x,y,z) \mid (x,y) \in D, \ u_1(x,y) \leq z \leq u_2(x,y) \} \)

\( \Rightarrow \) Show Figure 2 in p. 1042 Stewart

Formula: (do \( dz \) first)

\[
\iiint f(x,y,z) \, dV = \\
E \\
(\text{then use } \S 15.3 \text{ to handle } \iiint \ldots \, dA)_{\text{Stewart}} \\
\text{(move vertical/horizontal lines)}
\]

* Type 2: Similarly:

\( x \) between graphs of 2 functions in \( y,z \) with domain \( D \)

\( E = \{ (x,y,z) \mid (y,z) \in D, \ u_1(y,z) \leq x \leq u_2(y,z) \} \)

\( \Rightarrow \) Show Figure 7 in p. 1044 Stewart

Formula: (do \( dx \) first)

\[
\iiint f(x,y,z) \, dV = \\
E \\
\]

* Type 3: Similarly

\( y \) between graphs of 2 functions in \( x,z \) with domain \( D \).
\[ E = \left\{ (x,y,z) \mid (x,z) \in D, \quad u_1(x,z) \leq y \leq u_2(x,z) \right\} \]

\[ \Rightarrow \text{ Figure 8 in p. 1044 Stewart} \]

**Formula:** (do dy first)

\[ \iiint_E f(x,y,z) \, dV = \iint_D \left( \int_{u_1(x,z)}^{u_2(x,z)} f(x,y,z) \, dy \right) \, dA \]

**Remarks:**
- Forget the name type 1, type 2, type 3. The point is to express one variable between graphs of two functions in the other two variables, take \( \int \ldots \, d(\text{thin one variable}) \), then

\[ \iint_D \ldots \, dA \]

- How to use the above formulas:
  1. Use picture + lines parallel to \( z \)-axis (or \( x \)-axis, \( y \)-axis)
  2. From given equations \( \Rightarrow \) figure out the inequalities describing your solid
  3. Usually (a) + (b) + lots of practice.
- Inequalities: in $\mathbb{R}^3$, a surface of the form $\star = \star$
  (eg: $y = x^2 + z^2$, $x^2 + y^2 + z^2 = 3$, ...) breaks the 3-d spaces in 2 pieces. One piece is given by $\star \leq \star$ and other piece by $\star \geq \star$.
  $\Rightarrow$ plug-in a random point to check if $\leq$ or $\geq$ holds.

Eg(a) Find $\int \int \int_E \sqrt{x^2 + z^2} \ dV$

where $E$ is bounded by the paraboloid $y = x^2 + z^2$ and the plane $y = 4$.
(Hint: do dy first)

b) Try to do dx first & identify the difficulties.
Eg: Evaluate \( \iiint_E z \, dV \) where \( E \) is the solid tetrahedron bounded by the 4 planes \( x = 0, \ y = 0, \ z = 0 \), and \( x + y + z = 1 \).

(Hint: roughly sketch \( E \), express one variable between the 2 functions in other 2 variables, also identify the domain \( D \) in those 2 variables.)
Eq: \( \text{Fill in} \); 
\[
\int_0^1 \int_0^{x^2} \int_0^y f(x, y, z) \, dz \, dy \, dx
\]
\[
= \int_?^? \int_?^? \int_?^? f(x, y, z) \, dx \, dz \, dy
\]
Applications of Triple Integrals

* Volume: \( \text{Vol}(E) = \iiint_E 1 \, dV \)

* Moments & center of mass for a 3-d object:
  (also called centroid)

  Solid object occupies region \( E \) with density function \( \rho(x, y, z) \). Then
  
  . total mass \( m = \)
  
  . Moment about the \( yz \)-plane:
    Now; take directed distance from the \( yz \)-plane

  Formula:
  \[ M_{yz} = \]

  Similarly; moment about the \( xz \)-plane
  \[ M_{xz} = \]

  Moment about the \( xy \)-plane
  \[ M_{xy} = \]
The center of mass (also called centroid) of $E$:

$$\left(\bar{x}, \bar{y}, \bar{z}\right) \text{ with}$$

$$\bar{x} = \frac{M_{yz}}{m} = \frac{1}{m} \iiint_E x \rho(x, y, z) \, dV$$

$$\bar{y} = \frac{M_{xz}}{m} = \frac{1}{m} \iiint_E y \rho(x, y, z) \, dV$$

$$\bar{z} = \frac{M_{xy}}{m} = \frac{1}{m} \iiint_E z \rho(x, y, z) \, dV$$

Eg: Find the center of mass of the solid of constant density bounded by: the cylinder $x = y^2$ & the planes $x = z, \, z = 0, \, x = 1$.

(Due to time constraint, just find $m$ and $\bar{x}$. You are referred to Eg 6 p. 1048 for $\bar{y}$ & $\bar{z}$.)