Case 2: D in type II

\[ y \text{ between horizontal lines } y = c \text{ and } y = d \]
\[ x \text{ between curves } x = h_1(y) \text{ and } x = h_2(y) \]

**Example:**

![Graph showing Case 2 and Not Type II examples](image)

**Not Type II**

*Test:* Check if any horizontal line cuts in.
Formula:
\[ D = \{ (x, y) | c \leq y \leq d, \quad h_1(y) \leq x \leq h_2(y) \} \]
\[ \iint_D f \, dA = \] 

General method (similar to type I) when using type II:
- Sketch D, make sure type II
- Move horizontal lines
  * first to touch \( \Rightarrow c \)
  * last \( \Rightarrow d \) (usually involve points of intersection)
  * left most \( \Rightarrow h_1(y) \)
  * right most \( \Rightarrow h_2(y) \) (could happen: \( h_1 \) \& \( h_2 \) more than 1 formulas \( \Rightarrow \) need to break the integral)
Eg: (when \( g_1, g_2 \) or \( h_1, h_2 \) given by more than 1 formula)

\[
p(x)
\]

\[
Q(x)
\]

\[
a \quad b
\]

Yes: type I, write down formula for \( \iint_D f(x,y) \, dA \)?

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Eg: Find \( \iint_D xy \, dA \) when \( D \) is bounded by the line

\( y = x - 1 \) & parabola \( y^2 = 2x + 6 \)

using 2 methods: (note: draw \( D \) & see \( D \) is of type I & II)

a) Regard \( D \) as type I.

b) Regard \( D \) as type II.
Solution
(continue the solution on this page)
So, remark: when \( D \) is of both type, the choice of formulas (say, do \( dy\,dx \) (type I) or \( dx\,dy \) (type II)) may simplify the problem.

Eg: \[ \int_0^1 \int_x^1 \sin(y^2) \, dy\,dx = ? \]

(Hint: draw \( D \) & switch to \( dx\,dy \))
3) General case: D neither type I nor II

Property: \( D = D_1 \cup D_2 \): no overlap other than boundary

Then \( \iint_D f(x, y) \, dA = \iint_{D_1} f \, dA + \iint_{D_2} f \, dA \)

Idea: break D

**How to break?**

\[ 
\begin{array}{c}
\text{Using } \iint \text{ to calculate area:} \\
\text{Area}(D) =
\end{array}
\]

\text{Why? Isn't this supposed to be some volume!!?}