Eq: Evaluate \[ \int_1^3 \int_1^5 \frac{\ln y}{xy} \, dy \, dx \] in 2 ways.

(I'll do dydx in the lecture, and leave dx\,dy to you. You'll need u-subst \( u = \ln y \) in both ways.)

Eq: Find volume of the solid \( S \) bounded by the paraboloid \( x^2 + 2y^2 + z = 16 \), the planes \( x = 2 \), \( y = 2 \), and the 3 coordinate planes.
Eq: Find $\int_{0}^{2} \int_{0}^{3} ye^{-xy} \, dy \, dx$ by changing the order $dy \, dx \rightarrow dx \, dy$ first

(which makes the problem simpler)

Eq: $R = \{(r, \theta) : 0 \leq r \leq 2, 0 \leq \theta \leq \pi\}$, find $\int_{R} r \sin^{2} \theta \, dA$
Stewart §15.3 Double & Iterated Integrals over General Domain (instead of rectangles)

- What we have learned: $R = \text{rectangle}

  1) \text{Def of } \iint_R f\,dA \text{ using Riemann sums.}

  2) \text{Change } \iint_R f\,dA \text{ to iterated } \int \int \text{ & calculate}

- Now in §15.3: general domain $D$

  1) Theoretical def of $\iint_D f\,dA$ (not really important)

    - Find rectangle $R$ containing $D$

    - Define $F(x,y) = \begin{cases} f(x,y) & \text{if } (x,y) \text{ in } D \\ 0 & \text{if } (x,y) \text{ in } R \text{ but not in } D \end{cases}$

    - Define: $\iint_D f\,dA = \iint_R F\,dA$
2)*** Change \( \int_D f \, dA \) to iterated \( \int_s f' \, ds' \) and calculate

Rule: 
- the outside integral \( \int_s \)
- the inside integral \( \int_s \)
- switching \( dx \, dy \leftrightarrow dy \, dx \) & \( \int_s \int_s \leftrightarrow \int_s \int_s \): more complicated & not always possible using one integral, see later

Case 1: \( D \) has type I < \( x \) between 2 vertical lines < \( y \) between 2 functions in \( x \)

eg:
Eq: **NOT type I:**

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**Explain:** when some "moving vertical line" cut in more than 1 segment

**Formula:** D of type I

\[ D = \{(x, y): a \leq x \leq b, \ g_1(x) \leq y \leq g_2(x)\} \]

\[ \iint_D f \, dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f \, dy \, dx \]

**Eq:** Evaluate \( \iint_D (x+2y) \, dA \); D: region bounded by \( y = 2x^2 \) and \( y = 1 + x^2 \)

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**General method when using type I:**
- sketch D, make sure it's type I
- Moving vertical lines:
  - First vertical line to touch D \( \Rightarrow \) a \( \frac{\text{usually involve points of intersection}}{\text{Last}} \)
  - \( \frac{\text{lowest point} \Rightarrow g_1(x)}{\text{Highest point} \Rightarrow g_2(x)} \) (could happen that \( g_1 \) or/and \( g_2 \))
  - \( \frac{\text{need more than 1 formula}}{\Rightarrow \text{break the integral}} \)
Solution:

$y = 2x^2$

$y = 1 + x^2$
Case 2: D in type II

\[ y \text{ between horizontal lines } y = c \text{ and } y = d \]

\[ x \text{ between curves } x = h_1(y) \text{ and } x = h_2(y) \]

Eg

Test: check if any horizontal line cuts in