Our section website:
visit www.math.ubc.ca/~dknguyen then click on your section (or google "Khoa Nguyen math ubc")

Common website: given in the above section page too.

Instructor: Khoa D. Nguyen
Office hours: 2:15-3:15 Monday, 4:30-5:30 Wednesday, and 5-6 Thursday at LSK 300B

Go through basic information (given in section page)

- Homework: weekly WebWork
- Quizzes: weekly on Fridays; the first 2 quizzes are on Friday Sep 16 and Friday Sep 30
- Only one midterm on Friday Oct 14
- Final: TBD (announced in October, hopefully)

Questions???

Homework: download the free online textbooks after today's lecture
Before we start: review at home
- basic functions: polynomials, exp, log, trig
- their derivative and ∫

§10.1 Cartesian Coordinates in Space

Notation:
\[ \mathbb{R}^2: \text{2-d plane} = \text{points with 2 coordinates} \ (a, b) \]
\[ x \text{-coord.} \quad y \text{-coord.} \]

\[ \mathbb{R}^3: \text{3-d space} = \text{points with 3 coordinates} \ (a, b, c) \]
\[ x \text{-coord.} \quad y \text{-coord.} \quad z \text{-coord.} \]

Keep this picture in mind:

briefly explain right-hand rule
(not really important)

the origin O has coord. (0,0,0)
Eg1: draw the point P with coord. \((a, b, c)\)

Eg2: for the above picture, circle the correct word:
1) If \(c < 0\) then P lies above/\(\bigcirc\) below the \(xy\)-plane.
2) If \(a < 0\) then P lies at the front/\(\bigcirc\) back of the \(yz\)-plane.

Eg3: describe the surfaces in \(\mathbb{R}^3\) represented by the given equations:

a) \(x = 0\) : the \(yz\)-plane
b) \(y = 0\) : the \(xz\)-plane
c) \(z = 0\) : the \(xy\)-plane
d) \(z = 3\) : the plane parallel to the \(xy\)-plane and 3-unit above it.
Eg 4: Type “plot \( z = x^2 + y^2 \)” in Wolfram Alpha

⇒ sometimes, pictures are produced in “boxes” instead of the typical picture

Q: Where is \((0,0,0)\) in this Wolfram Alpha picture?

Eg 5: Intersect the surface \( z = x^2 + y^2 \) in Eg 4 with the planes

a) \( z = 1 \)

b) \( z = 4 \)

the plane \( c) \ z = 9 \)

1) The shape of these intersections?

2) Sketch the surface.

Answer:

1) a) \[ \begin{cases} x^2 + y^2 = 1 \\ z = 1 \end{cases} \]

A circle of radius 1 on the plane \( z = 1 \)

c) \[ \begin{cases} x^2 + y^2 = 9 \\ z = 9 \end{cases} \]

A circle of radius 3 on the plane \( z = 9 \)

d) \[ \begin{cases} x^2 + y^2 = 4 \\ z = 4 \end{cases} \]

A circle of radius 2 on the plane \( z = 4 \)
Distance & Sphere

* Recall $\mathbb{R}^2$

\[
\| \overline{P_1P_2} \| = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}
\]

(also length of segment $\overline{P_1P_2}$)

(other authors use $P_1P_2$ instead of $\overline{P_1P_2}$ for line segment, also use $\| \|$ instead of $\| \|$ for length)

Eq: $P_1(-1,3), P_2(2,1)$

\[
\| \overline{P_1P_2} \| = \sqrt{(-1-2)^2 + (3-1)^2}
= \sqrt{13}
\]

Circle: equation representing circle of radius $r$ with center $C(a,b)$?

Answer: equation represents points $P(x,y)$ such that $\|PC\| = r$

\[
\sqrt{(x-a)^2 + (y-b)^2} = r
\]

or

\[
(x-a)^2 + (y-b)^2 = r^2
\]
For $\mathbb{R}^3$: similar formula

- Distance $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$

$$||P_1P_2|| = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

- Eq representing the sphere with radius $r$ centered at $C(a, b, c)$ is: start with $P(x, y, z)$, need $||PC|| = r$

$$\sqrt{(x-a)^2 + (y-b)^2 + (z-c)^2} = r$$

or

$$(x-a)^2 + (y-b)^2 + (z-c)^2 = r^2$$

Eg 7: What does the eq represent?

a) $x^2 + y^2 + z^2 = 1$
   Ans: the sphere of radius 1 center $(0, 0, 0)$

b) $z = \sqrt{1-x^2-y^2}$
   Ans: the upper hemisphere of radius 1 center $(0, 0, 0)$ (since $z \geq 0$)

(How about $z = -\sqrt{1-x^2-y^2}$)

Similarly: the lower hemisphere of $r = 1$, center $(0, 0, 0)$ since $z \leq 0$
c) \((x-1)^2 + (y-2)^2 + (z+1)^2 \leq 5\)

Ans: If you were given "=5" instead of "\leq 5", you got the sphere of surface

Here, due to "\leq 5", you actually get the sphere and every point inside

⇒ the solid sphere of radius \(\sqrt{5}\) center \((1,2,-1)\)

d) \(3 \leq (x-1)^2 + (y-2)^2 + (z+1)^2 \leq 5\)

Ans: on or inside the sphere of \(r=\sqrt{5}\), center \((1,2,-1)\) \(\Rightarrow\) in between and

AND on or outside the sphere \(r=\sqrt{3}\), center \((1,2,-1)\) \(\Rightarrow\) in between the 2

spheres.

e) \(x^2 + y^2 + z^2 - 2x - 6y + 8z = 15\)

(Hint: review the trick to "complete the square":)

- \(x^2 + 2x + 1 \Rightarrow (x+1)^2\)
- \(x^2 - 10x + 25 \Rightarrow (x-5)^2\)
- \(x^2 + ax + \frac{a^2}{4} \Rightarrow (x + \frac{a}{2})^2\)

Answer:

\[
\begin{align*}
(x-1)^2 + (y-2)^2 + (z+4)^2 & = 36 \\
(x-1)^2 + (y-2)^2 + (z+4)^2 & = 36
\end{align*}
\]

⇒ the sphere of radius 6 center \((1,2,-4)\).