Problem 1. Let \( \mathbb{N}_0 := \mathbb{N} \cup \{0\} \). We consider a function \( f : \mathbb{N} \rightarrow \mathbb{N}_0 \) satisfying the following properties:

(a) for any \( m, n \in \mathbb{N} \), we have that \( f(m + n) - f(m) - f(n) \in \{0, 1\} \)
(b) \( f(2) = 0 \);
(c) \( f(3) > 0 \); and
(d) \( f(9999) = 3333 \).

Compute \( f(2019) \).

Problem 2. Find all real numbers \( a \) for which the equation

\[
16x^4 - ax^3 + (2a + 17)x^2 - ax + 16 = 0
\]

has 4 distinct real roots which form a geometric progression.

Problem 3. Let \( P(x) \) be a monic polynomial of degree 3 with integer coefficients. If one of its roots equals the product of the other two roots, then prove that there exists an integer \( m \) such that

\[
2P(-1) = m \cdot (P(1) + P(-1) - 2 - 2P(0)).
\]

Problem 4. Let \( m, n \in \mathbb{N} \). In a box there are \( m \) white balls and \( n \) black balls. We extract randomly two balls from the box; if the two balls have different colors, then we put back in the box a white ball, while if the two balls have the same color, then we put back in the box a black ball. We repeat this procedure until there is left in the box only one single ball. What is the probability that this last ball is white?