Math 340 Lecture 27

**Phase I for the dual simplex method.** So now, if we start out with a dictionary and it’s feasible (constants \(\geq 0\)), we go on with the (primal) simplex method. If it’s dual feasible (coefficients of the objective function \(\leq 0\)) we can now go with the dual simplex method. But what if it’s neither? Example: our initial dictionary is

\[
\begin{align*}
z &= -2x_1 + x_2 + x_3 \\
x_4 &= -3 + x_1 - x_2 + x_3 \\
x_5 &= 1 + x_1 - x_2 + x_3 \\
x_6 &= -2 + x_2 - 2x_3
\end{align*}
\]

In this case it’s neither feasible nor dual-feasible, so we still need a “phase I” step. Idea here: change to an auxiliary problem by changing objective something that is dual feasible - all coefficients \(\leq 0\). Standard choice is \(-x_1 - x_2 - x_3\), so our auxiliary problem is

\[
\begin{align*}
z' &= -x_1 - x_2 - x_3 \\
x_4 &= -3 + x_1 - x_2 + x_3 \\
x_5 &= 1 + x_1 - x_2 + x_3 \\
x_6 &= -2 + x_2 - 2x_3
\end{align*}
\]

Our dictionary is now dual-feasible so we can go with the dual simplex method! I’ll try to do this just with working with the original dictionary here. Dual simplex method says we should let \(x_4\) leave (largest negative constraint), and then \(x_1\) enters (remember we’re looking for positive coefficients in the matrix and comparing them to the objective function), so we get

\[
\begin{align*}
z' &= -3 - x_1 - x_2 - 2x_3 \\
x_1 &= 3 + x_4 + x_2 - x_3 \\
x_5 &= 4 + x_4 - x_3 \\
x_6 &= -2 + x_2 - 2x_3
\end{align*}
\]

and then \(x_5\) leaves (only negative constant) and \(x_2\) enters (only positive coefficient) and we get

\[
\begin{align*}
z' &= -7 - x_1 - 2x_6 - 4x_3 \\
x_1 &= 5 + x_4 + x_6 + x_3 \\
x_5 &= 4 + x_4 - x_3 \\
x_2 &= 2 + x_6 + 2x_3
\end{align*}
\]

So we’ve done some pivots on our auxiliary primal dictionary (via the dual simplex method, so corresponding to actual reasonable pivots in the dual dictionary) and gotten to a feasible solution! Now we can go back to the original problem with this dictionary, and write out the objective function \(z = -2x_1 + x_2 + x_3\) in terms of our new dictionary:

\[
\begin{align*}
z &= -8 - 2x_4 - x_6 + x_3 \\
x_1 &= 5 + x_4 + x_6 + x_3 \\
x_5 &= 4 + x_4 - x_3 \\
x_2 &= 2 + x_6 + 2x_3
\end{align*}
\]

Now we proceed with the usual (primal) simplex method like always, letting \(x_3\) enter and \(x_5\) leave to get

\[
\begin{align*}
z &= -4 - 2x_4 - x_6 - x_5 \\
x_1 &= 9 + 2x_4 + x_6 - x_5 \\
x_3 &= 4 + x_4 - x_5 \\
x_2 &= 10 + 2x_4 + x_6 - 2x_5
\end{align*}
\]

And then we’re done, and have the optimal solution.