LECTURE 1 (1/3/17)

MATH 221 - MATRIX ALGEBRA SECTION 204.

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COURSE WEBSITE www.math.ubc.ca/~dcollins/math221

QUIZZES EACH TUESDAY AT BEGINNING OF CLASS

NO HOMEWORK TURNED IN.
SUGGESTED PROBLEMS FOR YOU TO WORK OUT
(ON WEBSITE).

GRADING BREAKDOWN:

10%	QUIZZES	(LOWEST TWO DROPPED)
20%	MIDTERM 1	(TUES JAN 31)
20%	MIDTERM 2	(TUES MAR 7)
50%	FINAL	

OFFICE HOURS: THURS 2:00-3:00 LSK 300
FRI 1:00-2:00

ADDITIONAL HELP: MATH LEARNING CENTER LSK 301/302

TEXTBOOK: LINEAR ALGEBRA AND ITS APPLICATIONS
DAVID LAY 3RD UBC EDITION.
(READ SECTIONS BEFORE CLASS!)

WHAT IS LINEAR ALGEBRA?

THE STUDY OF SYSTEMS OF LINEAR EQUATIONS

SIMPLE EXAMPLE: BUY 4 POUNDS OF BANANAS, PAY \$2?

EACH POUND OF BANANAS COSTS 50 CENTS.

LINEAR EQUATION $4x = 2 \longrightarrow x = 2/4 = 1/2$.

MORE COMPLICATED: BUY 4 POUNDS OF APPLES AND 2 POUNDS OF TOMATOES AND PAY \$14

LINEAR EQUATION: $4x + 2y = 14$

POSSIBLE SOLUTIONS:

$x = 3 \quad y = 1$

$x = 2 \quad y = 3$

$x = 1 \quad y = 5$

BUT: IF MY FRIEND BUYS 1 POUND OF EACH AND PAID \$5

LINEAR EQUATION: $x + y = 5$

ONLY SOLUTION TO BOTH: $x = 2 \quad y = 3$

SYSTEM OF LINEAR EQUATIONS:

$$\begin{aligned} 4x + 2y &= 14 \\ x + y &= 5 \end{aligned}$$

(2 VARIABLES AND 2 EQUATIONS).

WHAT WE WANT TO STUDY:

- DETERMINE HOW TO SOLVE SYSTEMS (ALGORITHMICALLY)
- DETERMINE WHEN SYSTEMS CAN BE SOLVED

$$\left.\begin{aligned} x+y&=2\\ x+y&=3 \end{aligned}\right] \text{ CAN'T SOLVE THIS!} \qquad (\text{EXISTENCE})$$

- DETERMINE WHEN SYSTEMS HAVE ONLY ONE SOLUTION

$$\left.\begin{aligned} x+y&=5\\ 2x+2y&=10 \end{aligned}\right] \text{ MANY SOLUTIONS!} \qquad (\text{UNIQUENESS})$$

- THEN: BETTER CONCEPTUAL INTERPRETATION.

FIRST: SOLVING SYSTEMS (ALGORITHMICALLY)

$$\begin{aligned} 4x+2y&=14\\ x+y&=5 \end{aligned}$$

ONE WAY: SOLVE FOR $x=5-y$ IN 2ND EQUATION.
PLUG IT INTO 1ST: $4(5-y)+2y=14$

$$20-4y+2y=14$$
$$6=2y$$
$$y=3$$

ANOTHER WAY: ADDING EQUATIONS TOGETHER
SO:

$$\begin{aligned} 4x+2y&=14\\ -2(x+y)&=-2\cdot(5)\\ \hline 2x+0y&=4 \end{aligned} \qquad 2x=4 \rightarrow x=2$$

THINK ABOUT THIS AS <u>MANIPULATING</u> OUR SYSTEM OF EQUATIONS

START: $4x + 2y = 14$
$x + y = 5$

$\Updownarrow$ REPLACE <u>EQUATION 1</u> BY <u>(EQUATION 1)</u> $- 2 \cdot$ (EQUATION 2)

$2x + 0y = 4$
$x + y = 5$

$\Updownarrow$ DIVIDE 1ST EQUATION BY 2.

$x = 2$
$x + y = 5$

$\Updownarrow$ REPLACING <u>EQUATION 2</u> BY (EQUATION 2) $-$ (EQUATION 1).

$x = 2$
$y = 3$

(AUGMENTED) MATRICES

COEFFICIENTS CONSTANTS

$$\begin{aligned} 4x + 2y &= 14 \\ x + y &= 5 \end{aligned} \quad \longleftrightarrow \quad \left[\begin{array}{cc|c} 4 & 2 & 14 \\ 1 & 1 & 5 \end{array}\right]$$

2x3 (AUGMENTED) MATRIX
(2 ROWS x 3 COLUMNS)

$\updownarrow$ ADDING -2 TIMES 2ND ROW TO 1ST.

$$\begin{aligned} 2x &= 4 \\ x + y &= 5 \end{aligned} \quad \longleftrightarrow \quad \left[\begin{array}{cc|c} 2 & 0 & 4 \\ 1 & 1 & 5 \end{array}\right]$$

$\updownarrow$ DIVIDE 1ST ROW BY 2

$$\begin{aligned} x &= 2 \\ x + y &= 5 \end{aligned} \quad \longleftrightarrow \quad \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 1 & 1 & 5 \end{array}\right]$$

$\updownarrow$ ADD -1 TIMES 1ST ROW TO 2ND.

$$\begin{aligned} x &= 2 \\ y &= 3 \end{aligned} \qquad \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 3 \end{array}\right]$$

ELEMENTARY ROW OPERATIONS (ON A MATRIX)

1. ADD A MULTIPLE OF ONE ROW TO ANOTHER.
2. MULTIPLY A ROW BY A NONZERO CONSTANT.
3. SWAP TWO ROWS

FACTS:
- EACH OF THESE ROW OPERATIONS IS INVERTIBLE (CAN BE UNDONE)
- DOESN'T CHANGE THE SOLUTIONS TO THE SYSTEM OF EQUATIONS IT CORRESPONDS TO.

GIVES SYMBOLIC WAY OF SIMPLIFYING SYSTEMS OF EQUATIONS.

EXAMPLE: (3 EQUATIONS, 4 VARIABLES). x_1, x_2, x_3, x_4

$$\begin{aligned} x_1 \quad\;\; - 2x_3 + 3x_4 &= -2 \\ 2x_1 + x_2 - x_3 \qquad\;\; &= 2 \\ x_1 + x_2 + x_3 - 4x_4 &= 5 \end{aligned}$$

$\updownarrow$ AUGMENTED MATRIX.

$$\left[\begin{array}{cccc|c} 1 & 0 & -2 & 3 & -2 \\ 2 & 1 & -1 & 0 & 2 \\ 1 & 1 & 1 & -4 & 5 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & -2 & 3 & -2 \\ 2 & 1 & -1 & 0 & 2 \\ 1 & 1 & -1 & -4 & 5 \end{array}\right]$$

$\updownarrow$ 3RD ROW − 1ST ROW

$$\left[\begin{array}{cccc|c} 1 & 0 & -2 & 3 & -2 \\ 2 & 1 & -1 & 0 & 2 \\ 0 & 1 & 3 & -7 & 7 \end{array}\right]$$

$\updownarrow$ 2ND ROW − 2·(1ST ROW)

$$\left[\begin{array}{cccc|c} 1 & 0 & -2 & 3 & -2 \\ 0 & 1 & 3 & -6 & 6 \\ 0 & 1 & 3 & -7 & 7 \end{array}\right]$$

$\updownarrow$ 3RD ROW − 2ND ROW

$$\left[\begin{array}{cccc|c} \underline{1} & 0 & -2 & 3 & -2 \\ 0 & \underline{1} & 3 & -6 & 6 \\ 0 & 0 & 0 & \underline{-1} & 1 \end{array}\right] \longleftrightarrow \begin{aligned} x_1 - 2x_3 + 3x_4 &= -2 \\ x_2 + 3x_3 - 6x_4 &= 6 \\ -x_4 &= 1 \end{aligned}$$

<u>ROW ECHELON FORM</u>

- EACH LEADING ENTRY TO THE <u>RIGHT</u> OF ENTRIES <u>ABOVE</u> IT
- (ROWS OF 0'S AT BOTTOM)

SOLUTION:

$x_4 = -1$

$x_1 = 2x_3 + 1$

$x_2 = -3x_3$

<u>x_3 CAN BE ANYTHING</u>

(FREE VARIABLE)

$$\left[\begin{array}{cccc|c} 1 & 0 & -2 & 3 & -2 \\ 0 & 1 & 3 & -6 & 6 \\ 0 & 0 & 0 & -1 & 1 \end{array}\right]$$

$$\Big\downarrow \text{ 3RD ROW} \cdot (-1)$$

$$\left[\begin{array}{cccc|c} 1 & 0 & -2 & 3 & -2 \\ 0 & 1 & 3 & -6 & 6 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right]$$

$$\Big\downarrow \begin{array}{l} \text{1ST ROW} - 3(\text{3RD ROW}) \\ \text{2ND ROW} + 6(\text{3RD ROW}) \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & -2 & 0 & 1 \\ 0 & 1 & 3 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right] \longleftrightarrow \begin{aligned} x_1 - 2x_3 &= 1 \\ x_2 + 3x_3 &= 0 \\ x_4 &= -1 \end{aligned}$$

REDUCED ROW ECHELON FORM (RREF)

ROW ECHELON FORM +

- EACH LEADING ENTRY IS 1.
- EACH COLUMN CONTAINING A LEADING ENTRY HAS ONLY 0'S ELSEWHERE.