• The quiz consists of six questions on seven pages.

• No calculators, electronic devices, notes, or books allowed.

• If you need more space for a problem, continue on the back of the page. But label clearly that your answer continues somewhere else.

• If you are row reducing a matrix, please write which row operations you’re using throughout the process.
Problem 1.  [15 points] Solve the matrix equation $Ax = b$ for

$$
A = \begin{bmatrix}
2 & 4 & 6 & 8 \\
3 & 6 & 10 & 13
\end{bmatrix} \quad b = \begin{bmatrix}
6 \\
10
\end{bmatrix}.
$$

Write your answer in parametric vector form.
Problem 2. [15 points] For the following linear transformations $\mathbb{R}^2 \to \mathbb{R}^2$ that are described geometrically, determine what each does to the standard basis vectors

$$e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

and then find the matrix of the transformation.

(a) $S : \mathbb{R}^2 \to \mathbb{R}^2$ given by first reflecting across the line $y = x$, and then reflecting across the $x$-axis.

(b) $T : \mathbb{R}^2 \to \mathbb{R}^2$ given by first applying the shear transformation

$$\begin{bmatrix} x \\ y \end{bmatrix} \mapsto \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x + y \\ y \end{bmatrix}$$

and then rotating by 90 degrees counterclockwise.
Problem 3. [15 points] Solve the inhomogeneous linear system $x_1 v_1 + x_2 v_2 + x_3 v_3 = w$ for the following vectors:

$$v_1 = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 3 \\ 0 \\ 3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix}, \quad w = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}.$$

Does this tell you that $w$ is or is not in the span of $v_1, v_2, v_3$?
Problem 4. [20 points] Each of the four parts of this problem describes a function. For each one, decide if the function is **linear** or **not linear**. If it is linear, write down the matrix corresponding to it. If it’s not linear, find two specific vectors $v, w$ such that you can show $T(v + w) \neq T(v) + T(w)$.

(a) $T : \mathbb{R}^3 \to \mathbb{R}^2$ given by

$$T \left( \begin{bmatrix} x \\ y \\ z \end{bmatrix} \right) = \begin{bmatrix} 3x + 2y + z \\ 4x + 3y + 2z + 1 \end{bmatrix}.$$ 

(b) $T : \mathbb{R}^2 \to \mathbb{R}^2$ where $T(v)$ is given by rotating $v$ by 180 degrees and multiplying the length by 3.
Problem 4 continued. (c) $T : \mathbb{R}^2 \to \mathbb{R}^2$ given by defining $T(v)$ to have the same direction as $v$ but length 1 longer than the length of $T(v)$. (And setting $T(0) = 0$).

(d) $T : \mathbb{R}^3 \to \mathbb{R}^3$ given by

$$T \left( \begin{bmatrix} x \\ y \\ z \end{bmatrix} \right) = \begin{bmatrix} x + y \\ y + z \\ z + x \end{bmatrix}.$$
Problem 5. [15 points] Suppose a linear transformation $T : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ satisfies

$$T\left(\begin{bmatrix} 2 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 5 \\ 5 \\ 6 \end{bmatrix} \quad T\left(\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right) = \begin{bmatrix} -2 \\ 4 \\ 0 \end{bmatrix}.$$ 

Solve for the entries of the $3 \times 2$ matrix

$$A = \begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix}$$

such that $T(v) = Av.$
Problem 6. [20 points] For which real numbers $h$ is the following list of vectors \textit{linearly dependent}, and for which $h$ is the list \textit{linearly independent}?

\[ v_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 2 \end{bmatrix} \quad \quad v_2 = \begin{bmatrix} 2 \\ -1 \\ -1 \\ 3 \end{bmatrix} \quad \quad v_3 = \begin{bmatrix} 3 \\ -2 \\ h \\ 5 \end{bmatrix} \]