Problems on differentiation rules

The following questions are intended to walk you through the establishment of several rules for differentiation. Although you likely know these rules already, it is less likely that you are familiar with the arguments that demonstrate why they work. In the pursuit of understanding, rather the simply the right answer, you must temporarily suspend this knowledge of how and exercise a skepticism that forces careful establishment of why. In the formal realm of mathematics, we are careful to proceed using only previously established facts. As you work through these questions, note how they proceed from simpler to more complicated cases, each relying on the previous to extend the scope of the rule.

1. Recall that we defined e to be the number for which
$$\lim_{h\to 0} \frac{e^h - 1}{h} = 1.$$
Using this fact, calculate the derivative of the function $f(x) = e^x$ at any point x_0.

2. Using the definition of the derivative, calculate the derivative of $a(t) = t^n$ where n is a positive integer. Notice that if n is not a positive integer, working from the definition is difficult.

3. Using the product rule, quotient rule, chain rule, implicit differentiation and the derivatives calculated above (but no other rules!), calculate the derivatives of the following functions. When you write down successive lines in your calculation, place the name of any rule you use in parenthesis next to the new line. Or if you use one of the derivatives above, put "as calculated in problem –" in parenthesis. The point here is that you are building these derivatives from the ground up with every step (and therefore each new rule) you write down explicitly justified by results that you have established earlier. In many of these, you will have to modify the equation before taking derivatives to put them in a form to which previous results can be applied.

   (a) $g(x) = e^{bx}$.

   (b) $h(x) = c^x$. The answer to the previous question along with some exponential and log algebra should be useful.

   (c) $k(x) = \ln(x)$.

   (d) $M(\alpha) = \arcsin \alpha$. If your answer still has a trig functions in it, you still have more simplifying to do. You might have to draw a triangle here.

   (e) $N(\alpha) = \arccos \alpha$. Same comment as above.

   (f) $b(t) = t^{-n}$ where n is a positive integer. Note that the rule you derived in problem 2 above, which looks similar, only applies when the power is a positive integer. Here we are trying to generalize the rule to the negative case.

   (g) $c(t) = t^{1/n}$ where n is an integer. Here we are trying to generalize the rule to the non-integer case.

   (h) $d(t) = t^{p/q}$ where p and q are both integers. Same comment as the previous problem.

   (i) $s(t) = t^w$ where w is irrational. Again, log algebra and differentiation rules will come in handy.