

Outline

- ▶ Modeling intro
- ▶ First order chemical kinetics.
- ▶ Second order chemical kinetics.

Modeling

- A model is an abstract representation of an object, system, or process, usually idealizing some features and neglecting others.



For a population with heritable variation in some characteristic that influences an individual's capacity to reproduce or success in reproduction, the frequency of those phenotypes that reproduce most successfully will increase.



$$\frac{\partial u}{\partial t} = D \nabla^2 u$$



Modeling and theories

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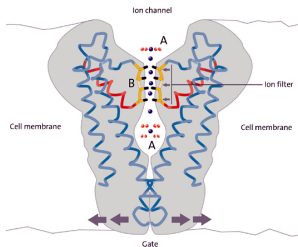
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- ▶ Example: Assume that the size of a population of bacteria grows at a rate proportional to its current size (our assumption). Translating into an equation, we get $dp/dt = rp$ (the assumption restated in our mathematical language). Solving this equation (an acceptable means of drawing conclusions), we find that the population grows exponentially. To determine if we've got a good theory, we must look at data.

Chemical kinetics

First order kinetics

- ▶ A reaction whose rate depends on the concentration of only one reactant.
- ▶ Example: an ion channel undergoing a conformation change (closed to open).

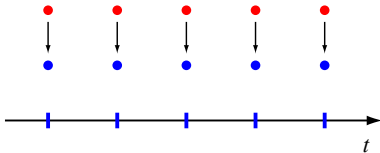


(From http://www.bio.miami.edu/~cmallery/150/memb/ion_channels.htm)

Chemical kinetics

First order kinetics

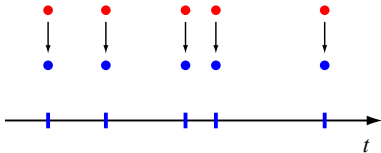
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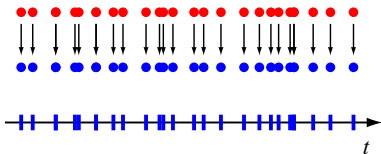
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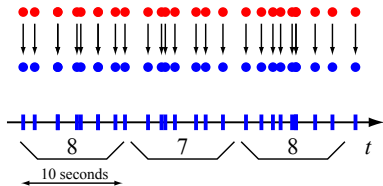
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Estimate of reaction rate: $\lambda = 7.7$ reactions per 10 seconds
per 100 molecules = $0.077 \text{ s}^{-1} \text{ molecule}^{-1}$

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- In the limit $\Delta t \rightarrow 0$,

$$\frac{dN}{dt} = -\lambda N.$$

Valid only for periods of time long enough for the average rate to give a good approximation of the number of reactions.

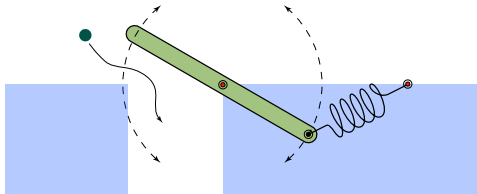
More molecules \Rightarrow short time intervals are ok.

- What determines λ , the average rate?

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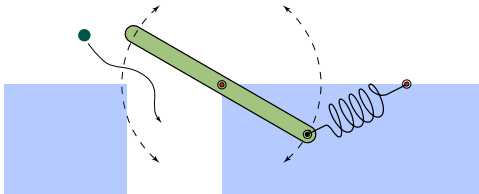
- Full protein structure too hard to analyze; build a simpler model...



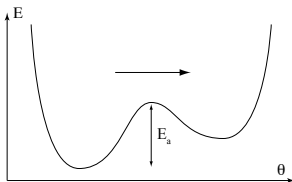
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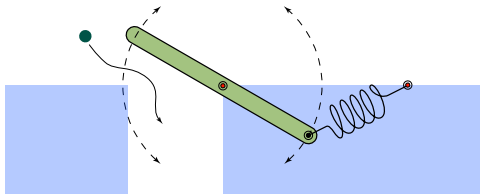
- ▶ Energy as a function of gate angle:



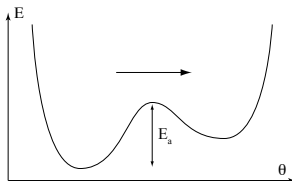
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- ▶ Average rate $\propto e^{-E_a/kT}$. Matlab movies...

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- ▶ The reaction rate equation:

$$\frac{dN}{dt} = -kN^2.$$

