Area between curves and related applications

- 1. A patient is administered a medication in such a way that the rate at which the concentration of this medication in the bloodstream changes is $r(t) = Ae^{-3t}$ (measured in milligrams per liter per hour $(mg/(L \cdot hr))$, with time (t) measured in hours).
 - (a) Find the concentration of the medication c(t) in the bloodstream at time t assuming that the initial concentration was zero.
 - (b) For the medication to take effect it must eventually reach a minimum concentration of 5 mg/L. However, above 10 mg/L it begins to have a toxic effect. Find the interval of effective and safe values for A.
 - (c) What is the earliest time at which the medication take effect without eventually reaching toxic levels?
- 2. A sink can hold a maximum of 20 liters before it overflows. Suppose the sink is initially empty. The inflow rate from the tap is given by $T(t) = A(1 + e^{-3t})$ and the outflow through the drain is $D(t) = A(1 e^{-4t})$. T(t) and D(t) are both measured in liters per minute and t is measured in minutes.
 - (a) How many liters of water are in the sink at time t?
 - (b) What is the maximum value of A for which the sink does not eventually overflow?
- 3. Below are graphs of the birth rate and death rate of a particular species of wild have in the arctic over a period of ten years starting in 1995 (x = 0). By examining the graphs, answer the following questions.



- (a) Determine the years in which the population size was increasing and in which years the population size was decreasing.
- (b) At the start of which year was the population of hares largest and when was it smallest? Briefly explain your answer.

Volumes of rotation

1. Calculate the volume of a vase which is formed by rotating the function $f(y) = 10+5\sin(\pi y/5)$ (measured in cm) between y = 0 cm and y = 10 cm about the y axis. Suppose a drop of water spills over the lip of the vase and drips all the way down the vase along its side to the table. Write down an expression for distance traveled by the drop from lip to base?



2. Suppose a bagel contains 1.5 calories per cm³. If slicing through the middle of the bagel (giving two U shaped pieces) reveals two circles of dough each of radius 1.5 cm and whose centers are 6 cm apart, how many calories are there in the bagel?



3. Find the volume of soup that will fit in a bowl with a height of 10 cm, bottom radius of 10 cm, top radius of 20 cm and whose sides are straight (i.e. lie on the surface of a cone). See figure.



Density of volumes of rotation

1. In the early stage of tumor growth, cancerous cells form a perfect sphere, approximately 4 mm in diameter. In the tumor shown below, the density of cells varies from left to right due to the presence of a blood vessel close to the left edge. The density of cells as a function of horizontal position is given by $\rho(x) = 6 - x$ where x is measured in millimeters and ρ is measured in thousands of cells.



- (a) Use the method of disks to find the volume of the tumor.
- (b) Calculate the total number of cells in the tumor.
- 2. A dam is built in a canyon so that the shape of the reservoir behind the dam is formed by rotating the parabola $y = \frac{x^2}{25} 100$ halfway about the y-axis; x and y are measured in meters. The dam is a straight wall and is built so that its top edge is at a height y = 0. Furthermore, the water contains sediment and the density of the sediment is a function of height with c(y) = -y/50 measured in kg/m³.
 - (a) What is the maximum volume of water that the reservoir can hold?
 - (b) How many kilograms of sediment is in the water when the reservoir is half full?



3. A cone-shaped coffee filter with a height of 10 cm and an opening radius of 10 cm is used to make coffee. While in use, the filter is full of water and the density of coffee grounds is given by $\rho(y) = -\frac{1}{10}y$ where y = 0 corresponds to the top of the filter. y is measured in cm and ρ is measured in grams/cm³.



- (a) Use the method of disks to find the volume of water in the filter.
- (b) Calculate the total mass of coffee grounds in the filter.