MATHEMATICS: ASSIGNMENT 3

Instructions:	No hand-written work. Submit using the assignment box in IBLC 361. Indicate subject (mathematics), assignment number, name and student ID# at the top of the front page. Multiple pages should be stapled; no folded-corner tricks, no paper clips.
Due:	Before 8:30 am, Tuesday, Oct 13.

1. Let $f(x)$ be a differentiable function everywhere on the real line. Let $g(x)$ be differentiable everywhere except at $x = a$. Give an example for each scenario below. Justify your claims.
   (a) Both $f \circ g$ and $g \circ f$ are differentiable everywhere on the real line.
   (b) $f \circ g$ is differentiable everywhere but $g \circ f$ is not.
   (c) $f \circ g$ is not differentiable everywhere but $g \circ f$ is.

2. Use a linear approximation to estimate each of the following: $\log(1.03)$, $0.0081^{1/3}$. Use a calculator to determine the relative error in your approximation.

3. Using a linear approximation of a function about a point p to approximate the value of the function at a point q is only a good idea when q is close to p. Unfortunately, this is a vague statement because "close" does not have an absolute meaning.
   (a) Define a function for which the linear approximation about x=0 provides a good approximation (within 1%) to the value of the function at x=0.1.
   (b) Define a function for which the linear approximation about x=0 provides a terrible approximation (off by more than a factor of 10) to the value of the function at x=0.1.
   (c) Define a function for which the linear approximation about x=0 provides a good approximation (within 1%) to the value of the function at x=1000.
   (d) In what sense is p "close to" or "far from" q in the examples above? Your answer should rely on some feature of f.

4. Show that the sum of the x- and y-intercepts of any tangent line to the curve $\sqrt{x} + \sqrt{y} = \sqrt{c}$ is equal to the constant c.

5. It has been hypothesized that the human heart is capable of beating only a fixed number of times and then it gives up. As far-fetched as this hypothesis may be, assume it to be true for the purpose of this problem.

   Oprah would like to develop a personal exercise regime that maximizes her lifespan. She discovers that by exercising every day for T hours, her heart rate throughout the rest of the day can be predicted by the formula:
$$H(T) = 2400\left(1 + \frac{1}{1+T}\right)$$
   where $H(T)$ is measured in beats per hour. During any exercise period, her heart rate is always 120 beats per minute.
   (a) Describe what $H(T)$ tells you about Oprah's heart's response to exercise.
   (b) Provide Oprah with some advice on her exercise regime.