

MATHEMATICS: ASSIGNMENT 1

Instructions:

Your answers must include justification for any steps that would not be obvious to a pre-calculus student. Hand-written work will not be accepted. You are free to use whatever typesetting/word-processing program you like but, for ease of typesetting mathematics, LaTeX is recommended. There will be a tutorial on installing and using LaTeX.

Due:

Monday, Sept 21.

1. Evaluate each of the following limits (if it exists):

(a) $\lim_{x \rightarrow \infty} \arctan(e^{1/x})$

(b) $\lim_{x \rightarrow 3} \frac{|x-3|}{x-3}$

(c) $\lim_{x \rightarrow \infty} \frac{x+3}{\sqrt{4x^2+1}}$

2. Let f , g and h be continuous functions satisfying the following equations:

$$\lim_{x \rightarrow 3} (f(x) + g(x) + h(x)) = 2,$$

$$\lim_{x \rightarrow 3} f(x) = -\lim_{x \rightarrow 4} g(x),$$

$$\lim_{x \rightarrow 3} \log(2g(x) + h(x)) = 0.$$

Let g be linear, with leading coefficient 1. Find $h(3)$.

3. Let

$$f(x) = \begin{cases} \frac{6a(x^2+1)}{2x^2+1} + \frac{b \log((x+1)^4)}{x} & \text{if } x < 0, \\ -2a - 4 & \text{if } x = 0, \\ \frac{a \sin x}{x} + b & \text{if } x > 0. \end{cases}$$

Find a and b such that f is continuous at $x = 0$.

4. Determine whether the following functions are continuous:

(a) $f \circ g \circ f \circ g \circ f$, where $f(x) = \begin{cases} 0 & \text{if } x < 1, \\ -\log x & \text{if } x \geq 1 \end{cases}$ and $g(x) = \cos x$

(b) the distance between Saturn and the planet closest to Saturn, as a function of time

5. Let

$$f(x) = \begin{cases} 0 & \text{if } x = 0 \text{ or } x \text{ is irrational,} \\ 1/q & \text{if } x = p/q, \text{ where } p \text{ and } q \text{ are coprime integers with } q > 0. \end{cases}$$

Find the numbers at which f is discontinuous.

Note: two integers are said to be *coprime* if their greatest common divisor is 1.