Today

- Summary of resonance
- Introduction to systems of equations
- Direction fields
- Eigenvalues and eigenvectors
- Finding the general solution (distinct e-value case)

Midterm comments

- Avg 83%

- Range 44-100%

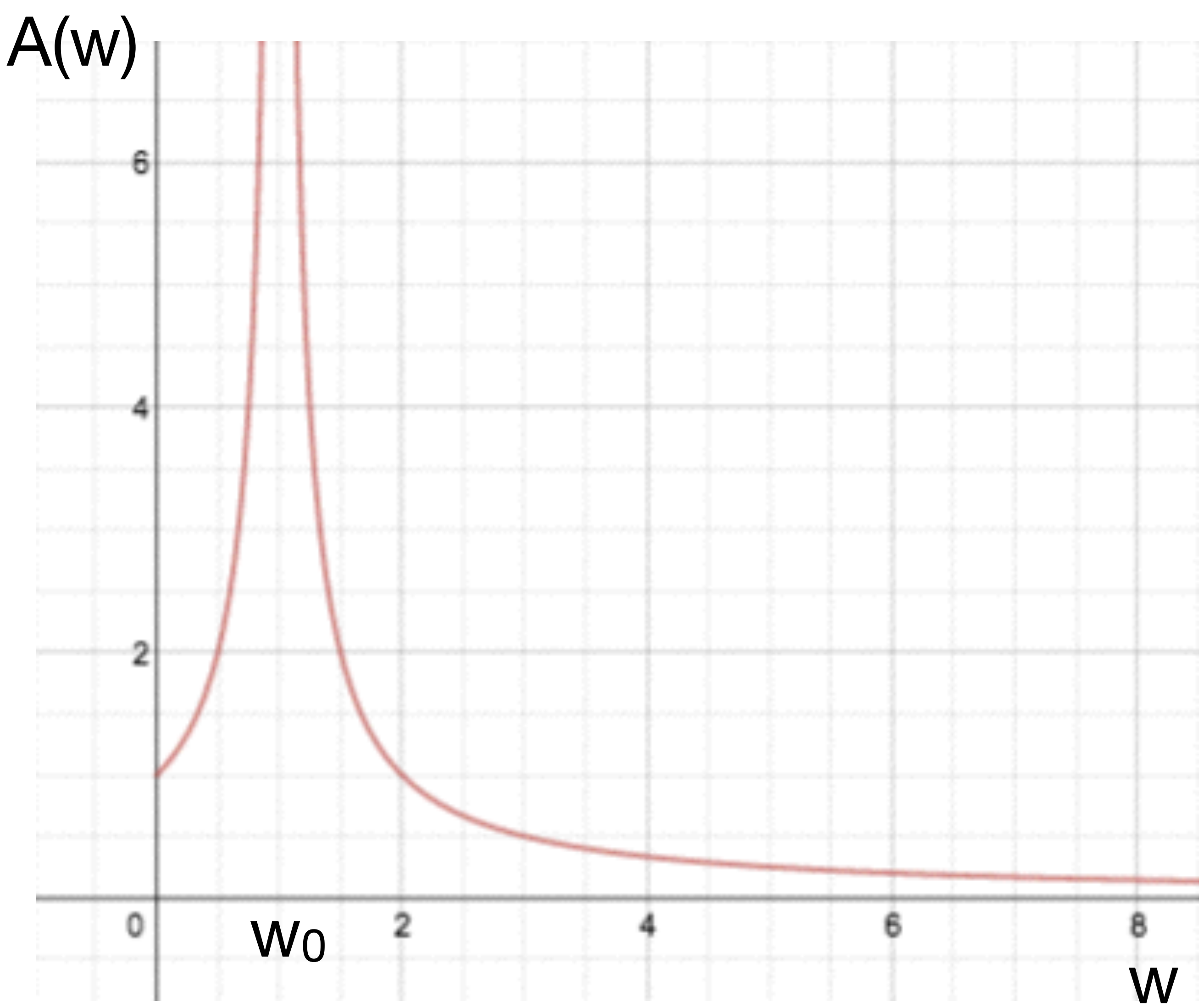
- Too easy; resonance.

- Learn log rules.

- Learn to check solutions.

Forced vibrations, no damping, summary

- Plot of the amplitude of the particular solution as a function of ω.



- Calculated:

$$A = \frac{F_0}{m(\omega_0^2 - \omega^2)}$$

- Plotted with:

$$\frac{F_0}{m} = 1, \;\; w_0 = 1$$

$$A(\omega) = \frac{1}{|\omega_0^2 - \omega^2|}$$

- Recall that for $\omega = \omega_0$, the amplitude grows without bound.

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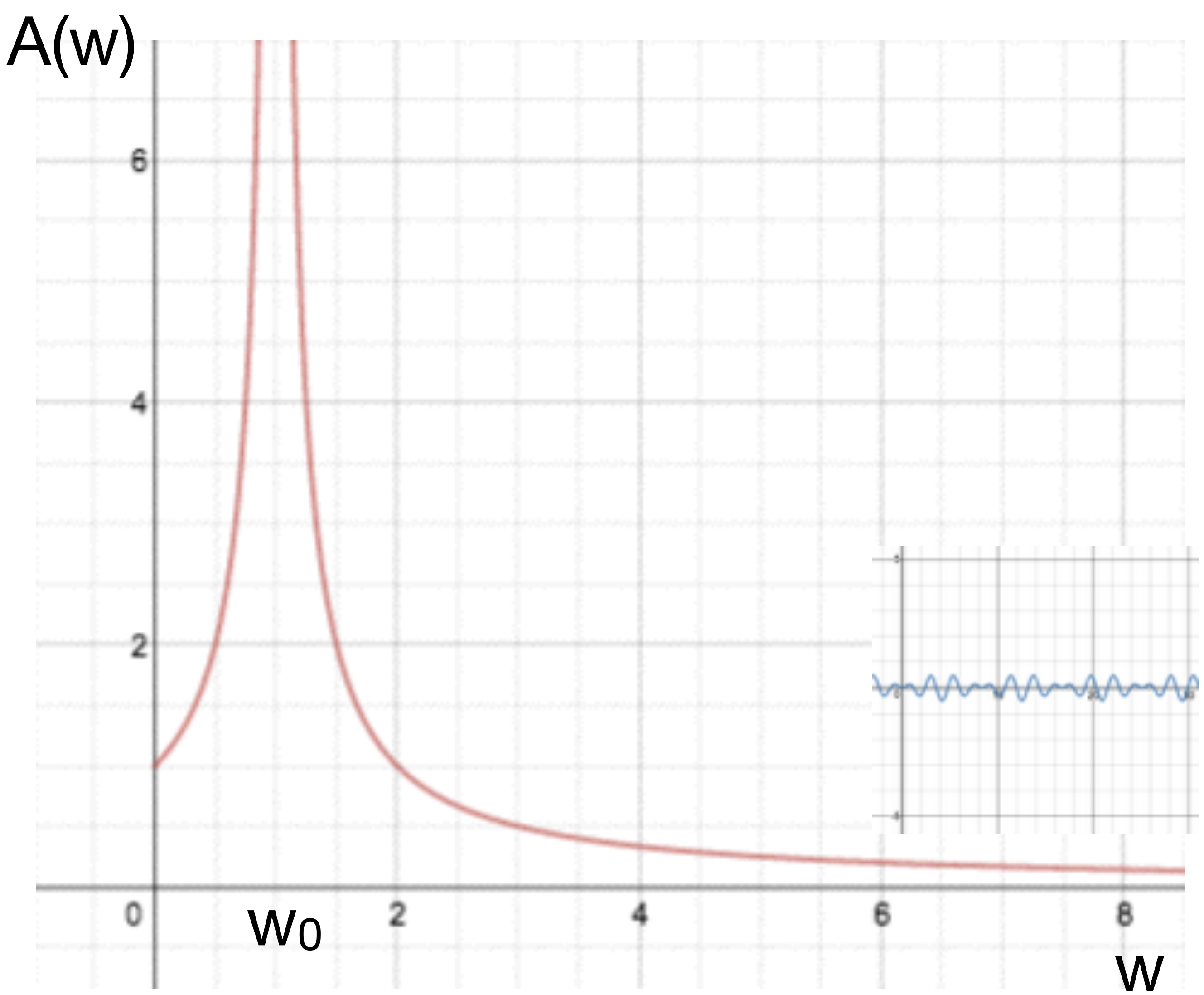

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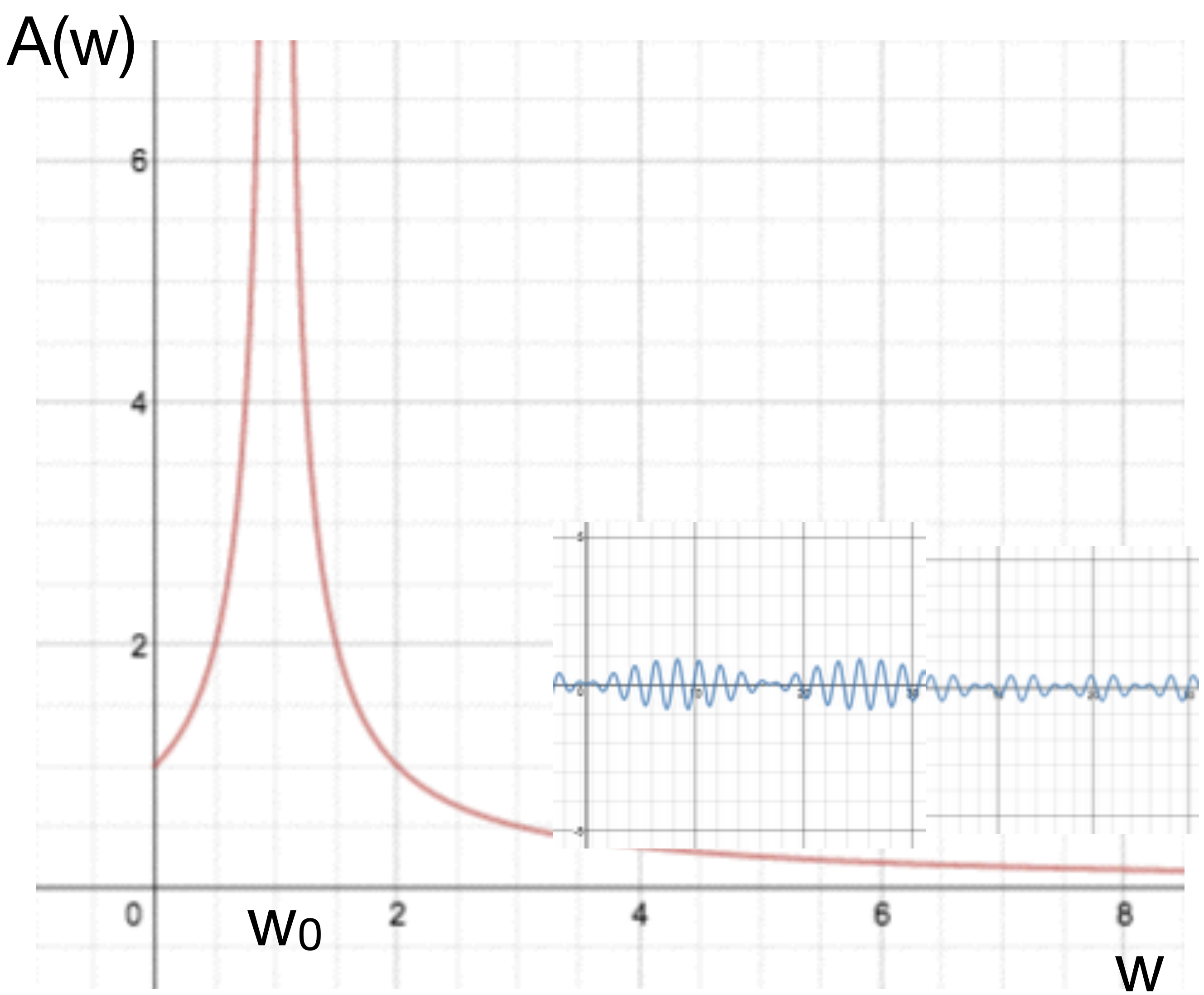

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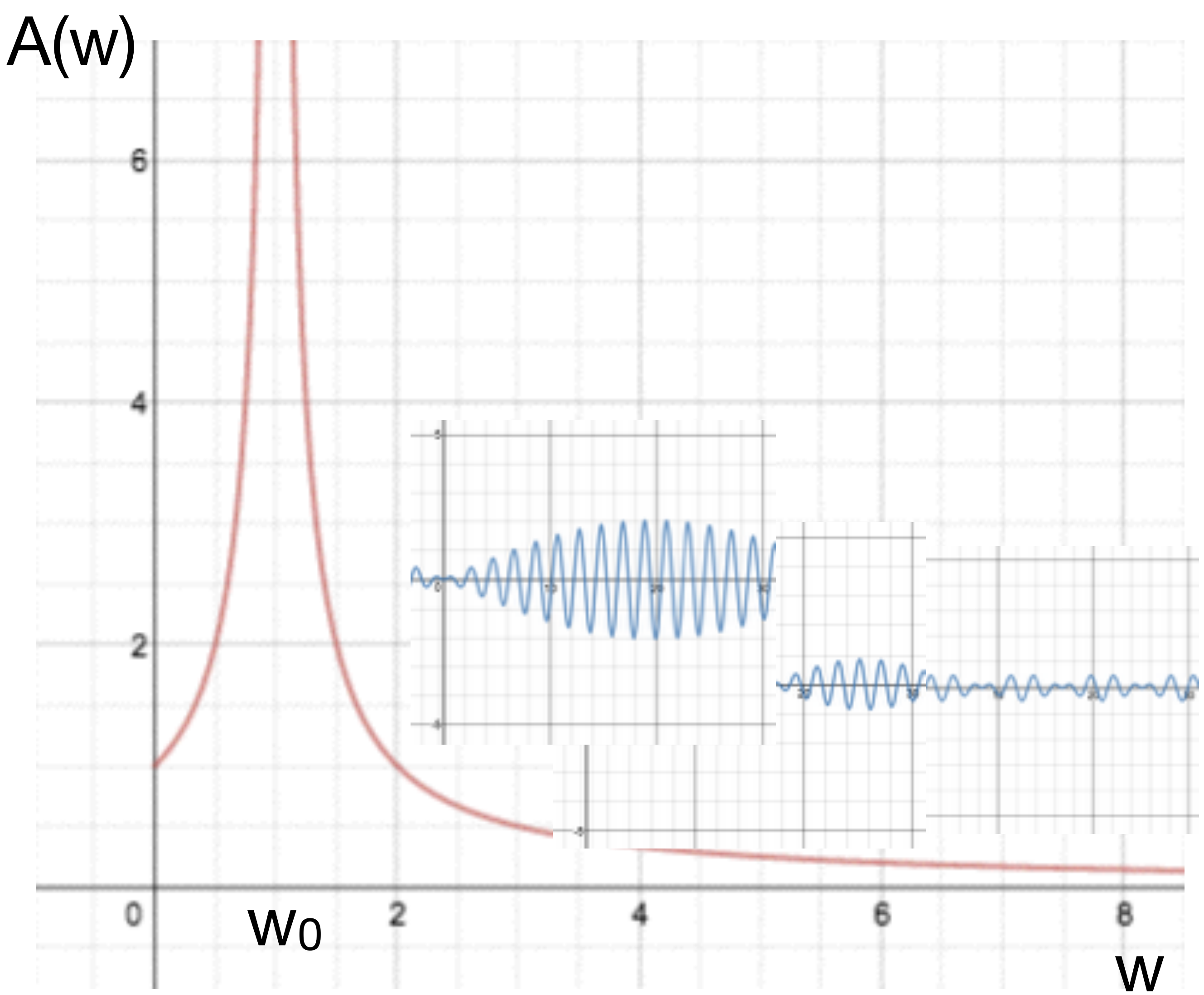

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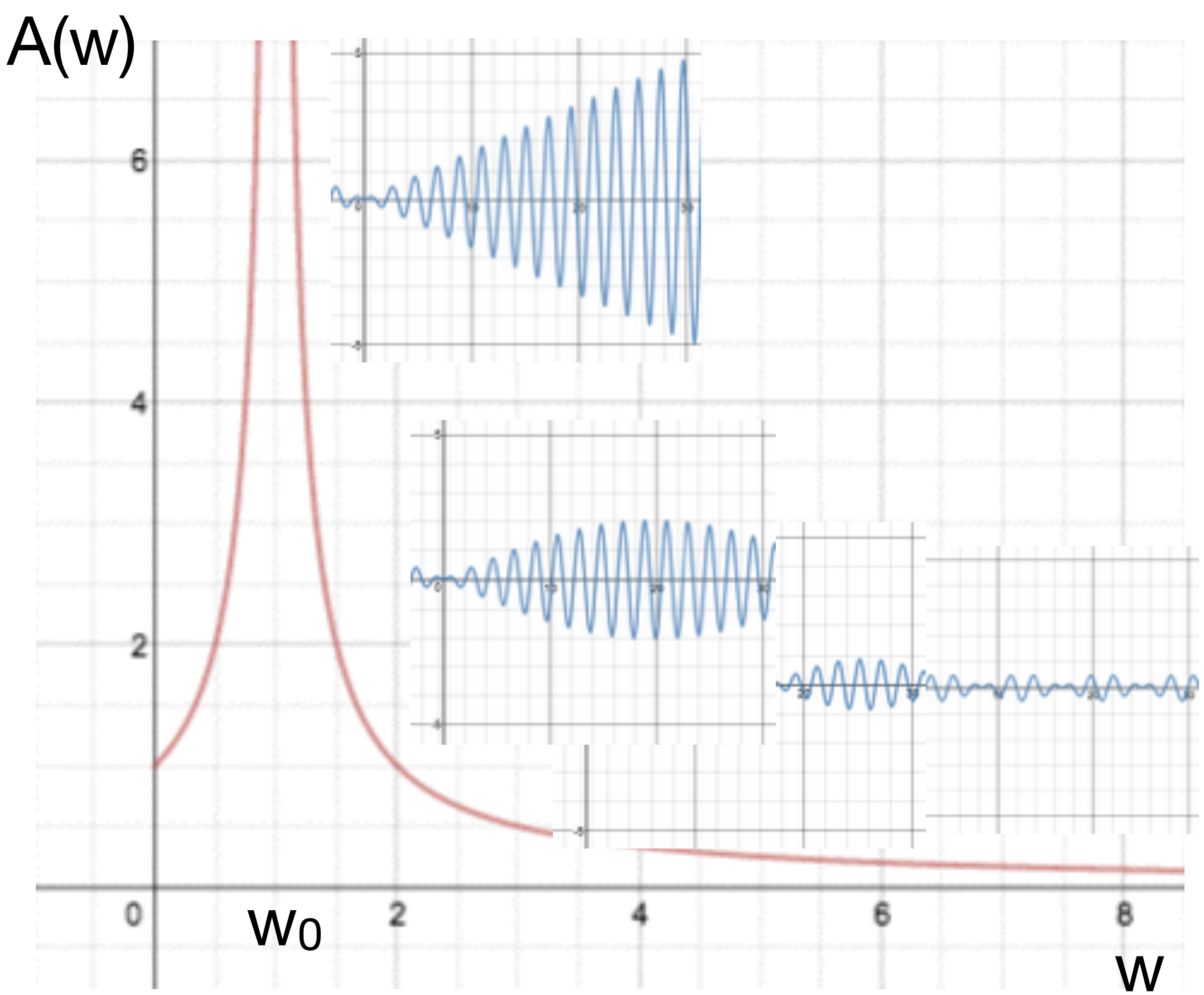

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Forced vibrations, with damping

$$m x'' + \gamma x' + kx = F_0 \cos \omega t$$

$$x'' + c x' + \omega_0^2 x = \frac{F_0}{m} \cos \omega t$$

No conflict with $x_h(t)$!

$$x_p = A \cos \omega t + B \sin \omega t$$

$$x_p' = -\omega A \sin \omega t + \omega B \cos \omega t$$

$$x_p'' = -\omega^2 A \cos \omega t - \omega^2 B \sin \omega t$$

$$-\omega^2 A \cos \omega t - \omega^2 B \sin \omega t + c(-\omega A \sin \omega t + \omega B \cos \omega t) + \omega_0^2 (A \cos \omega t + B \sin \omega t) = \frac{F_0}{m} \cos \omega t$$

$$\underbrace{(-\omega^2 A + c\omega B + \omega_0^2 A)}_{F_0/m} \cos \omega t + \underbrace{(-\omega^2 B - c\omega A + \omega_0^2 B)}_{0} \sin \omega t = \frac{F_0}{m} \cos \omega t$$

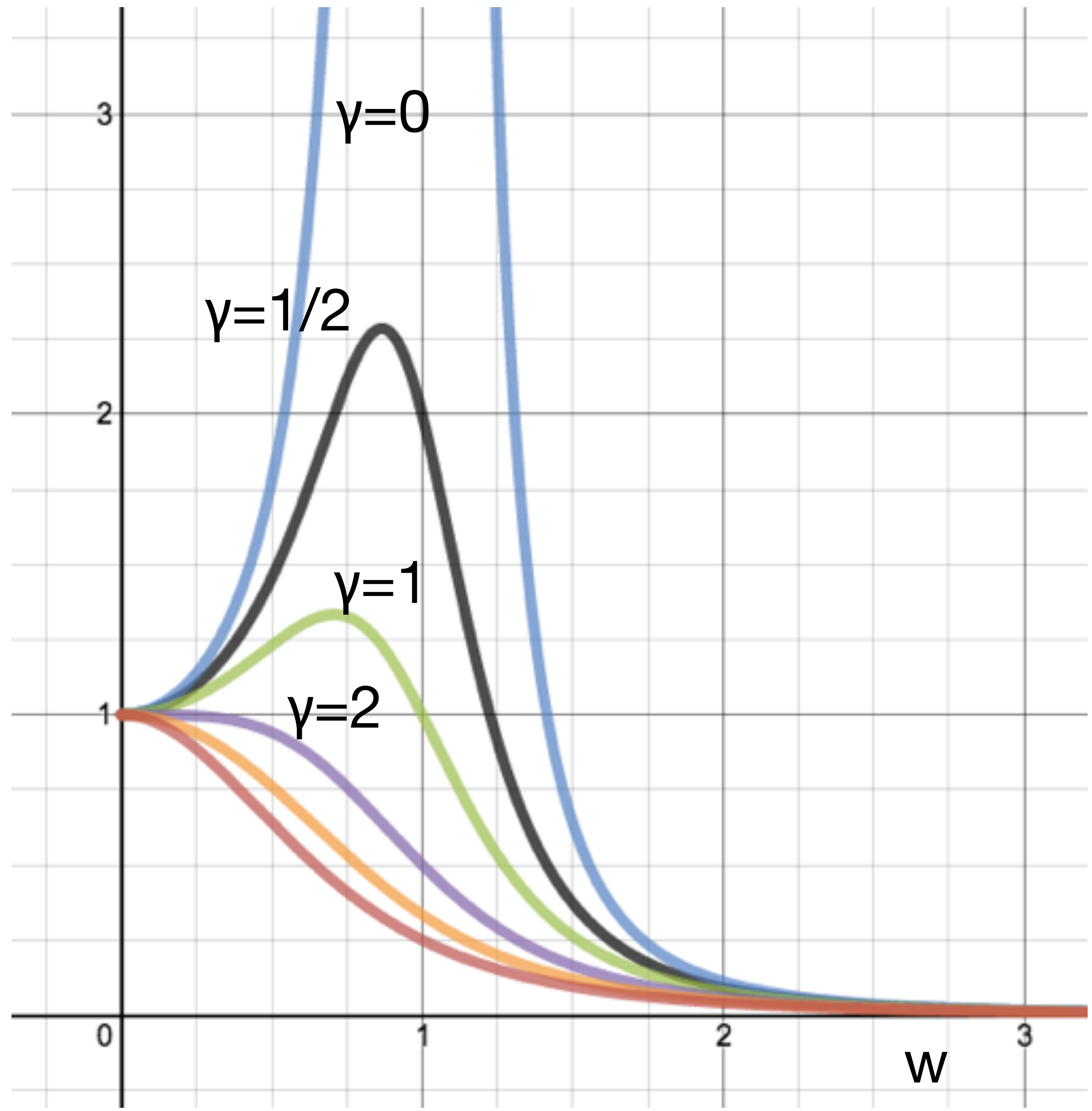
$$A = \frac{F_0}{m} \frac{\omega_0^2 - \omega^2}{(c\omega)^2 + (\omega_0^2 - \omega^2)}$$

$$B = \frac{F_0}{m} \frac{c\omega}{(c\omega)^2 + (\omega_0^2 - \omega^2)}$$

$$x_p(t) = \frac{F_0}{m} \cdot \frac{1}{\sqrt{(c\omega)^2 + (\omega_0^2 - \omega^2)^2}} \left(\frac{(\omega_0^2 - \omega^2)}{\sqrt{(c\omega)^2 + (\omega_0^2 - \omega^2)^2}} \cos \omega t + \frac{c\omega}{\sqrt{(c\omega)^2 + (\omega_0^2 - \omega^2)^2}} \sin \omega t \right)$$

Forced vibrations, with damping

Amplitude of solution



$$\text{Amp} = \frac{F_0}{m\sqrt{(c\omega)^2 + (\omega_0^2 - \omega^2)^2}}$$

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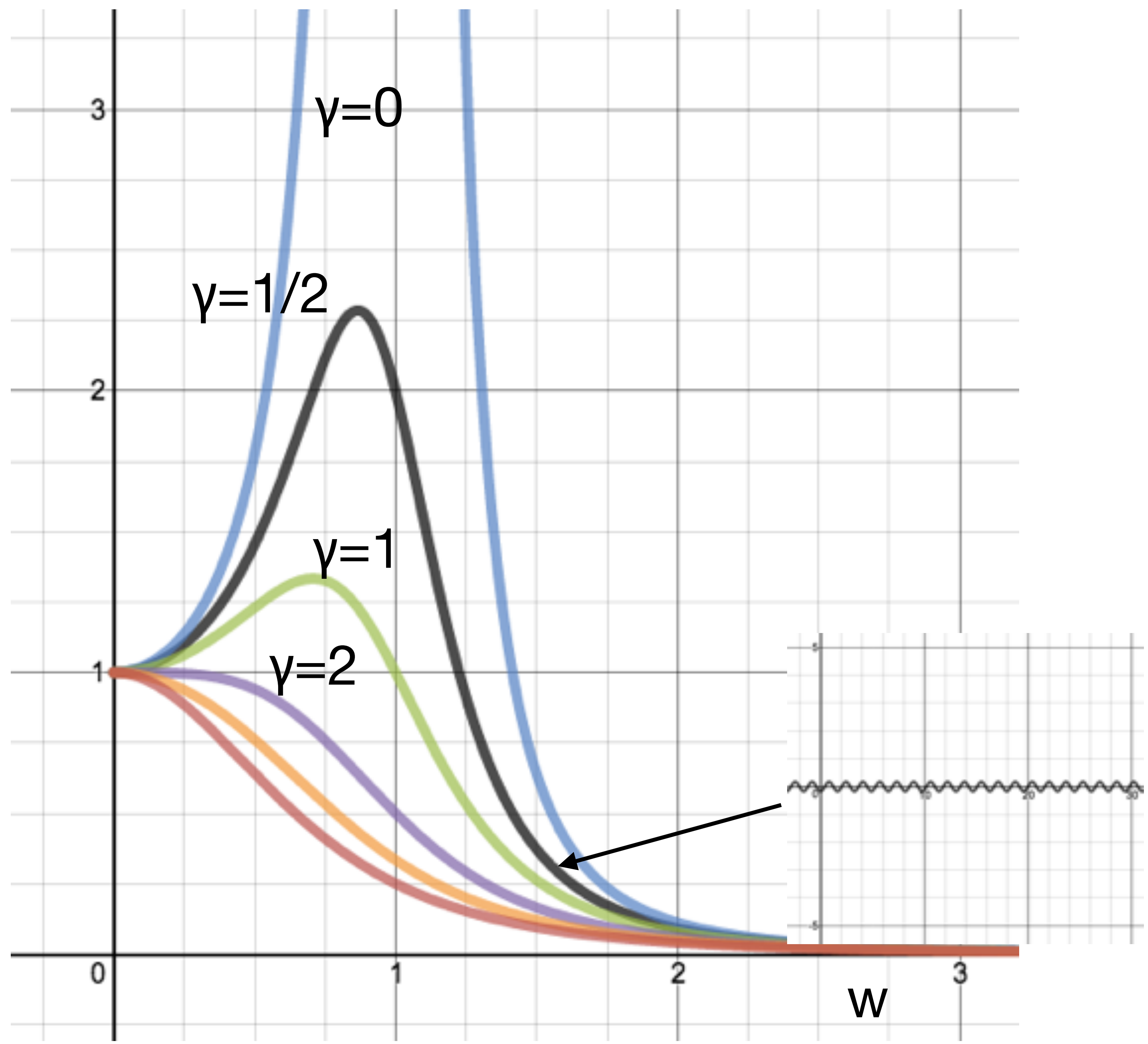

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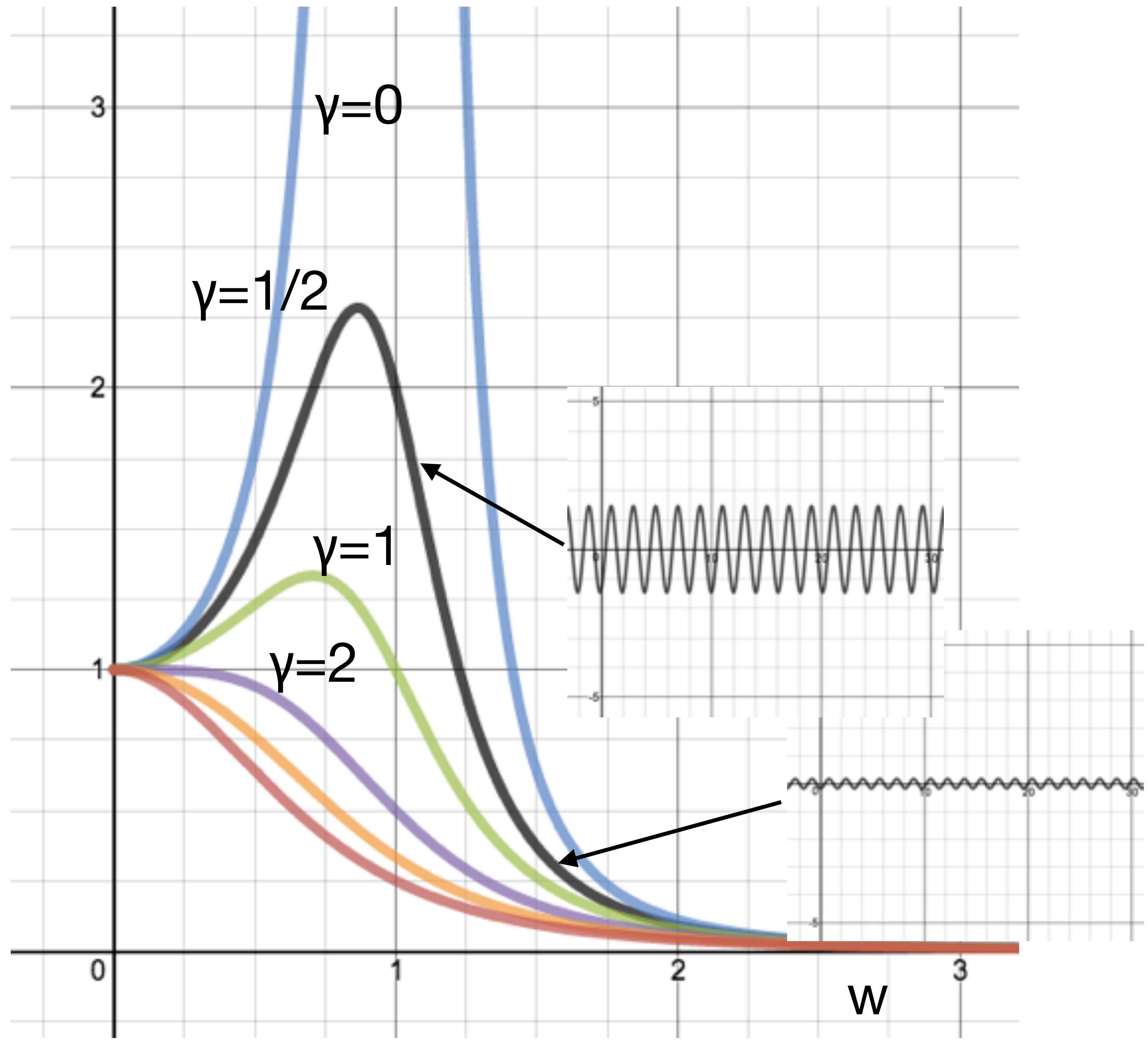

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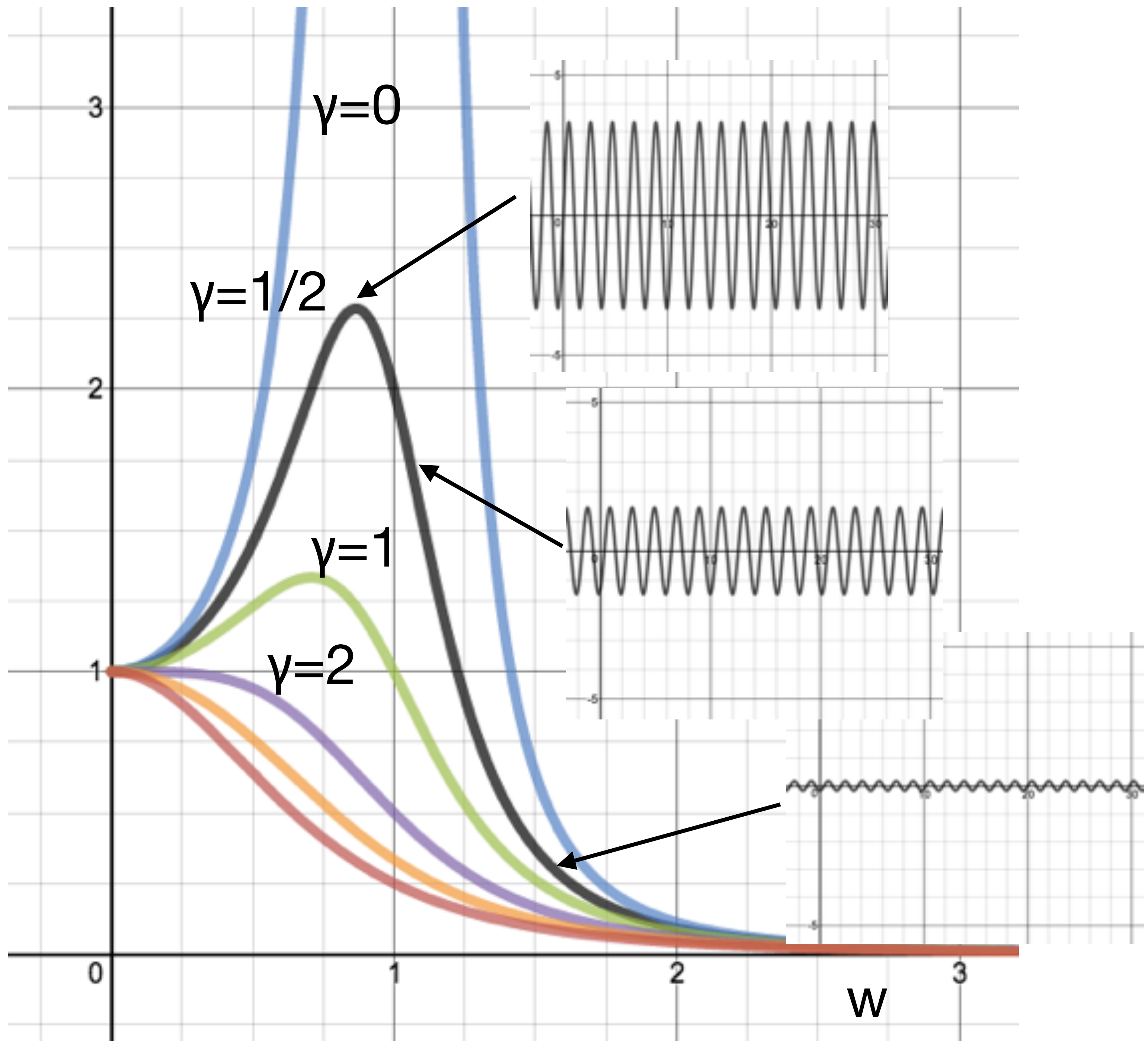

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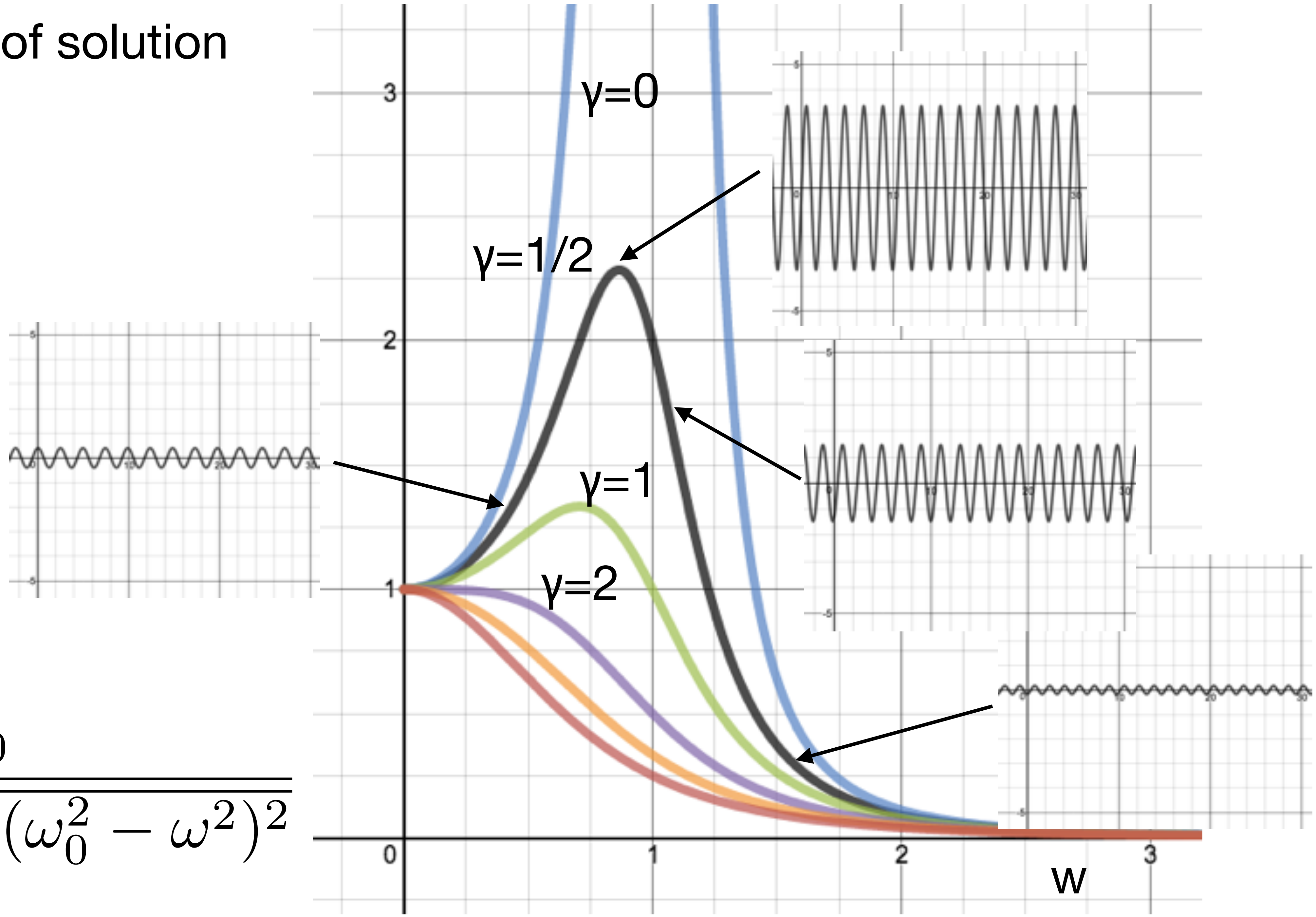

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$$\begin{aligned} x' &= v \\ v' &= -\frac{k}{m}x - \frac{\gamma}{m}v \end{aligned}$$

$$\begin{pmatrix} x \\ v \end{pmatrix}' = \begin{pmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{\gamma}{m} \end{pmatrix} \begin{pmatrix} x \\ v \end{pmatrix}$$

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    - populations of two species (e.g. predator and prey).

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- We'll focus on the case in which the matrix has constant entries. And homogeneous. For example,

$$\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$

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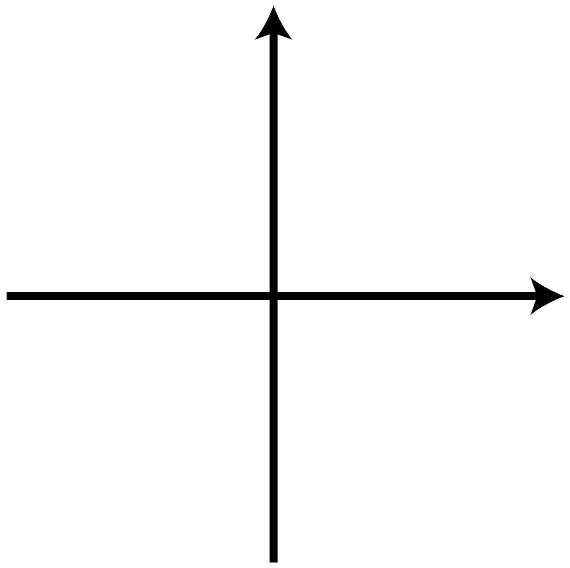

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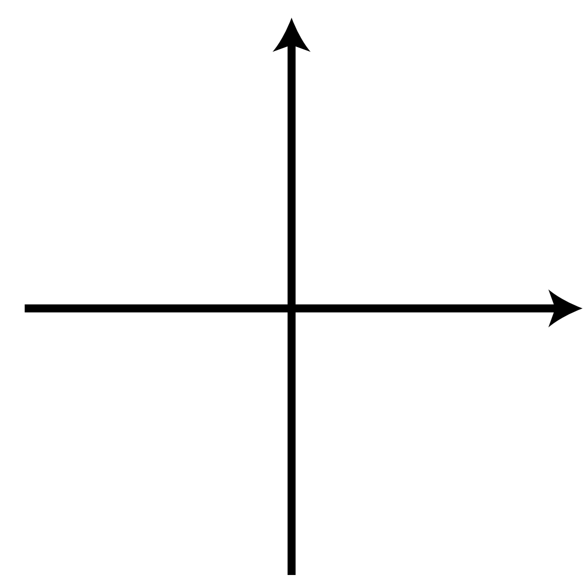

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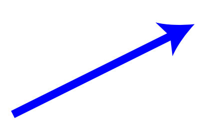

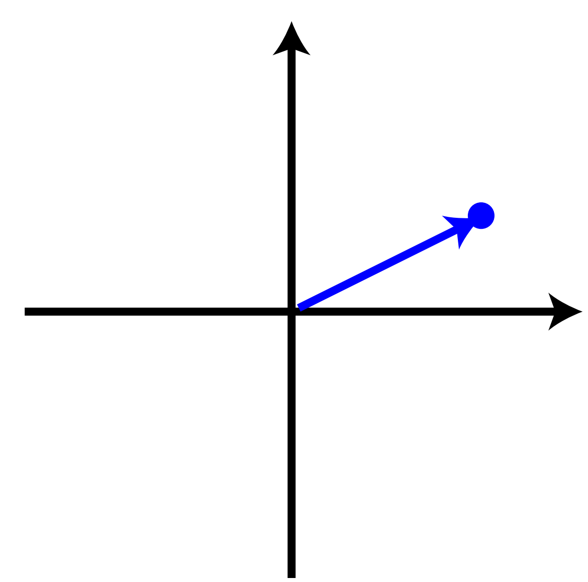

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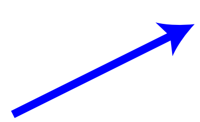
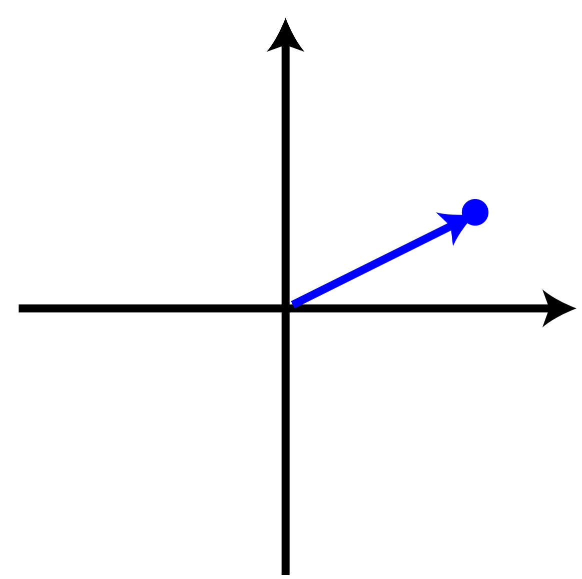


$$A\mathbf{x} = \begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix}\begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 9 \end{pmatrix}$$



Introduction to systems of equations

- Geometric interpretation - direction fields.

$$\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} \qquad \text{or} \qquad \mathbf{x}' = A\mathbf{x}$$

- Think of the unknown functions as coordinates $(x(t), y(t))$ of an object in the plane.
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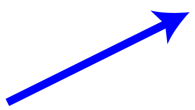

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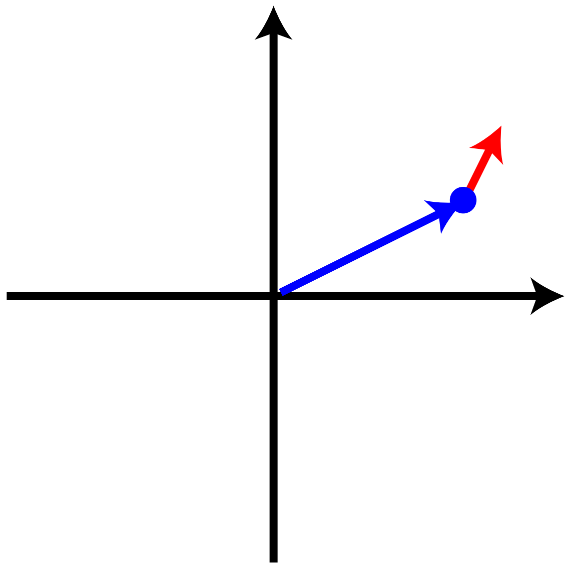

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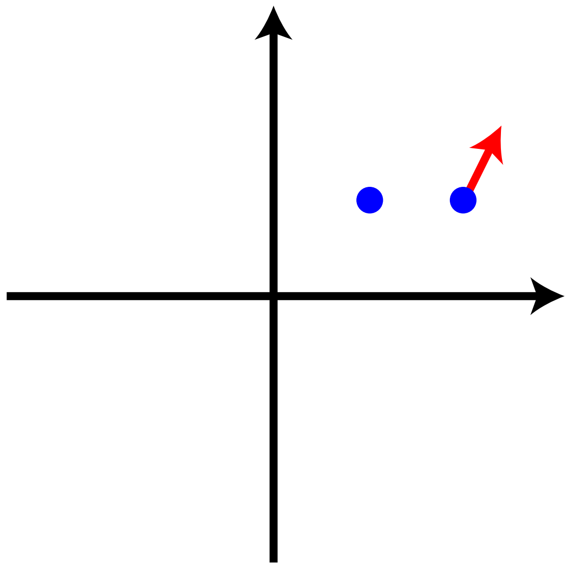

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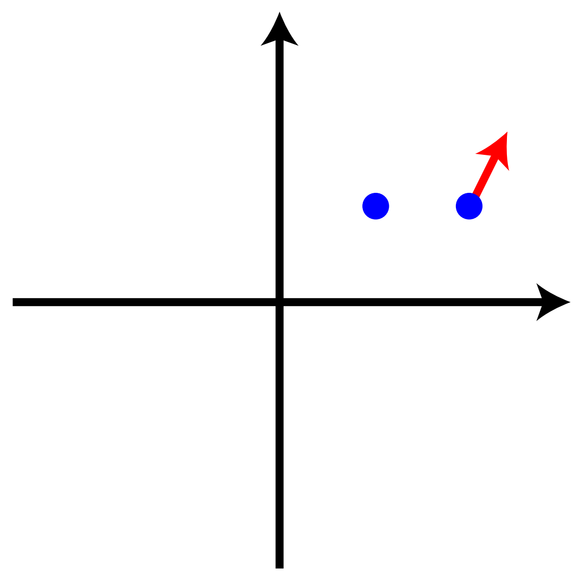

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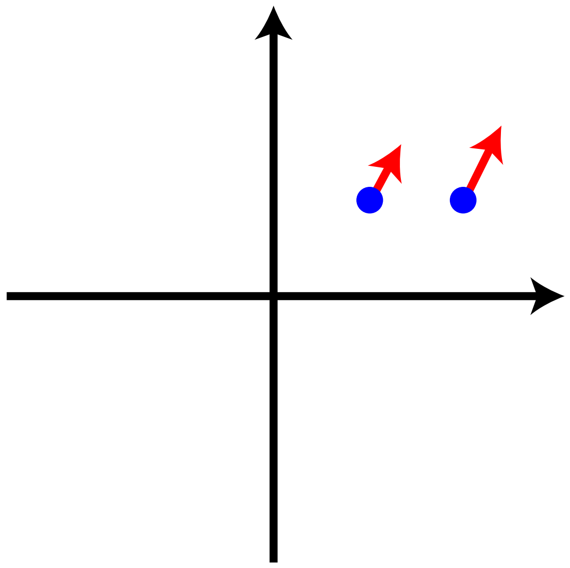

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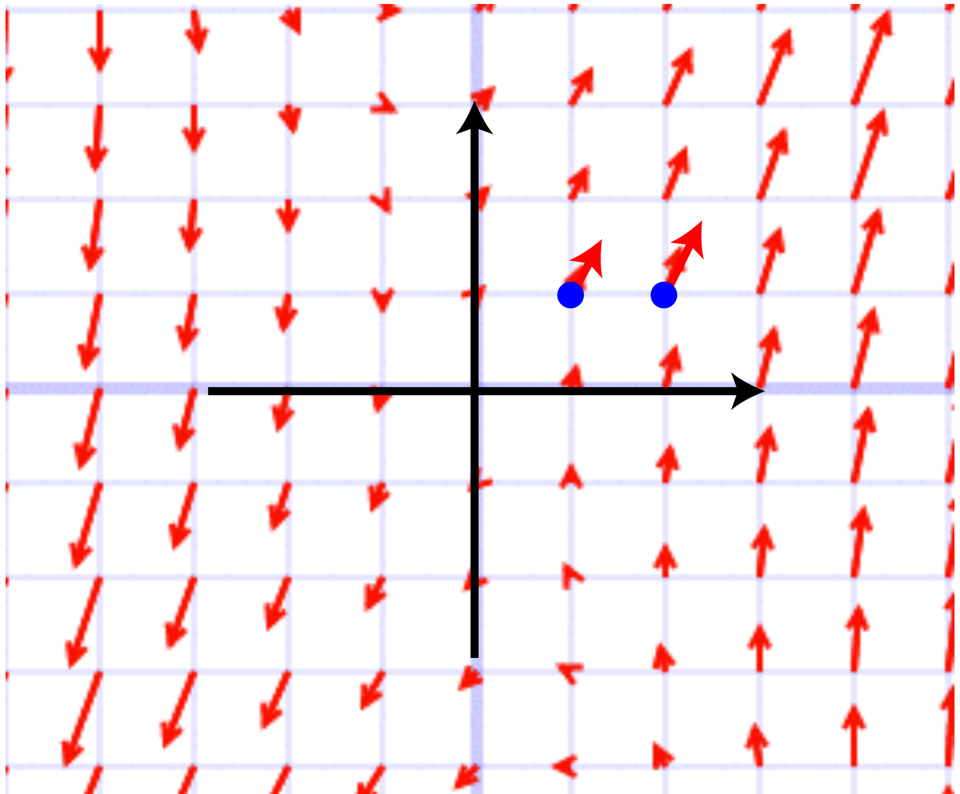

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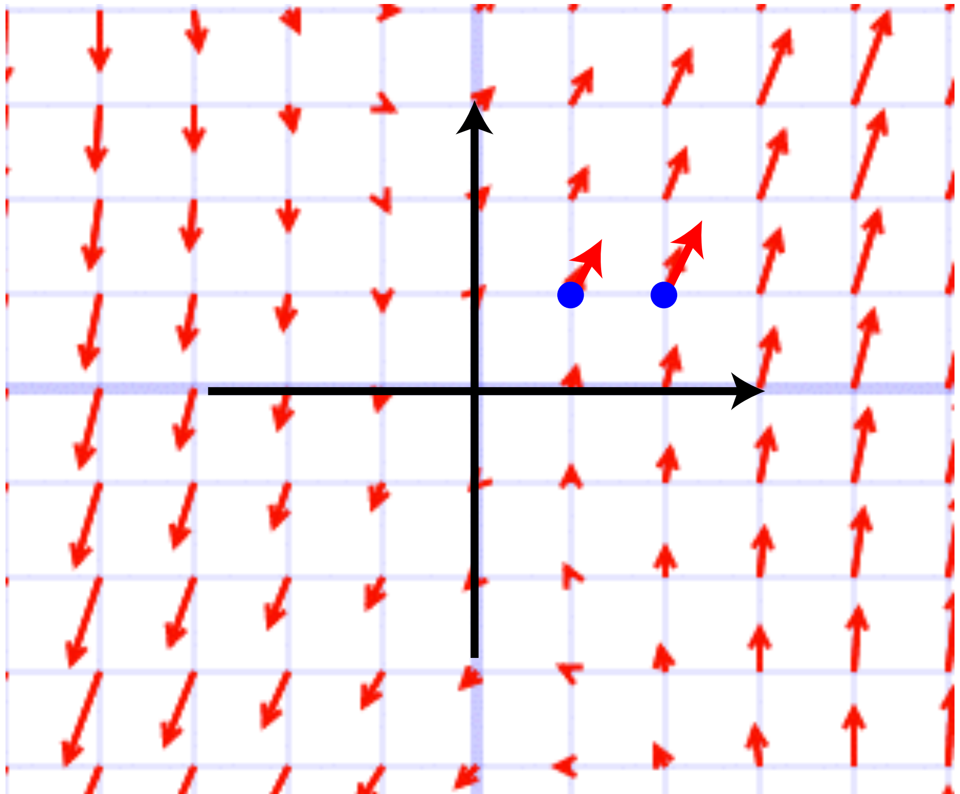

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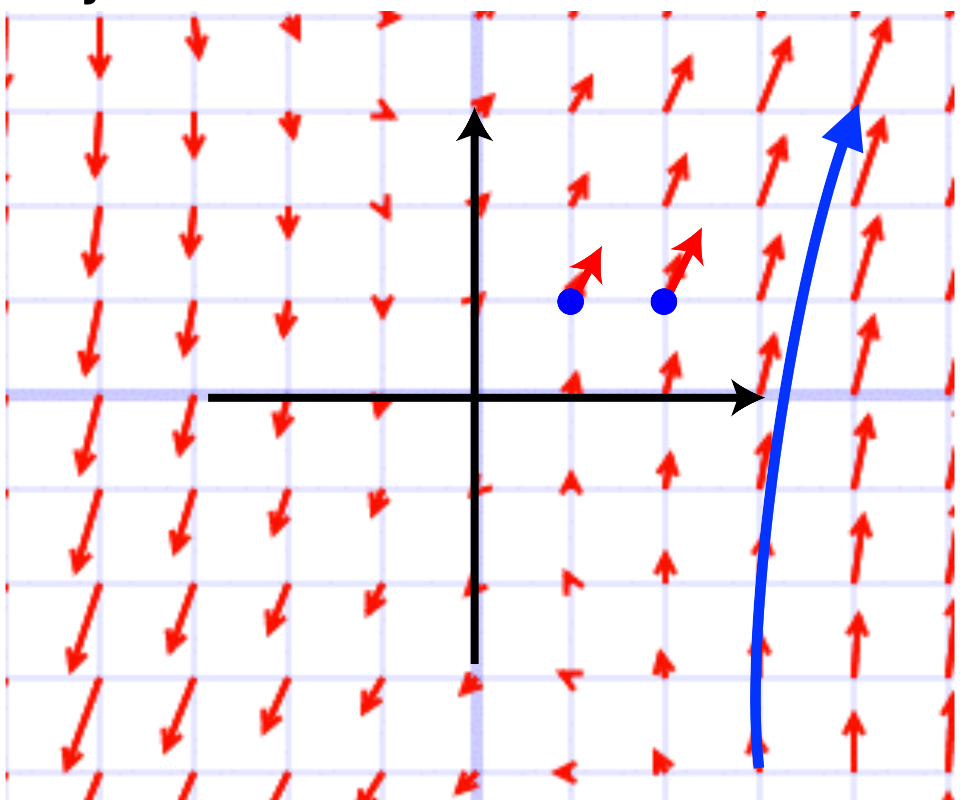

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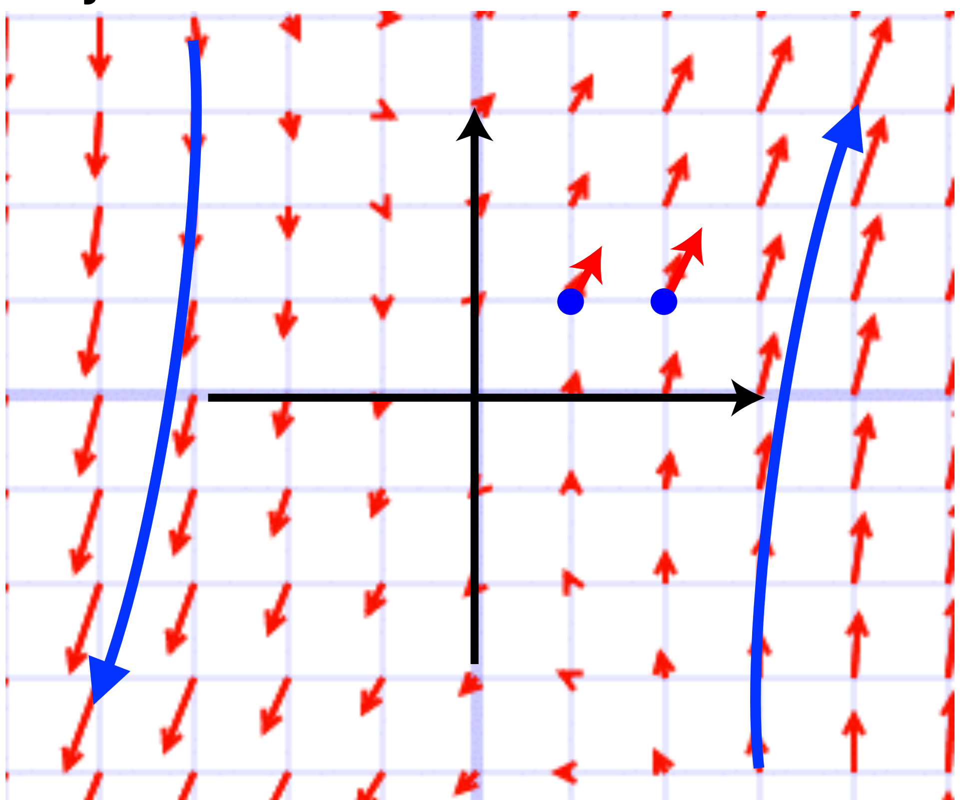

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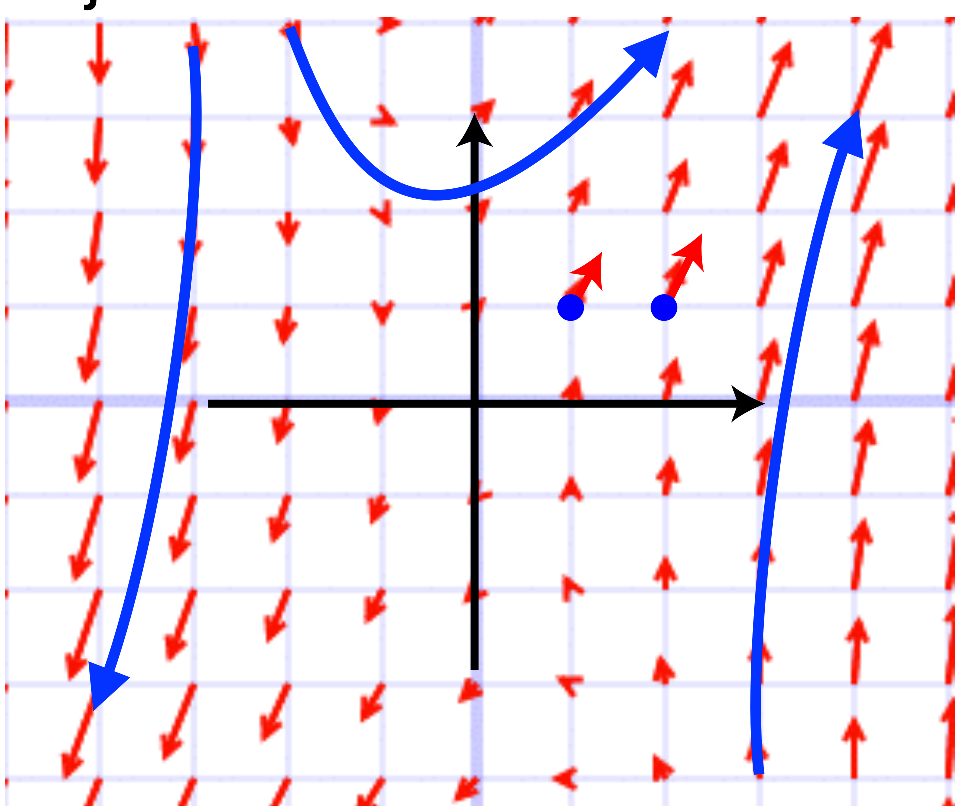

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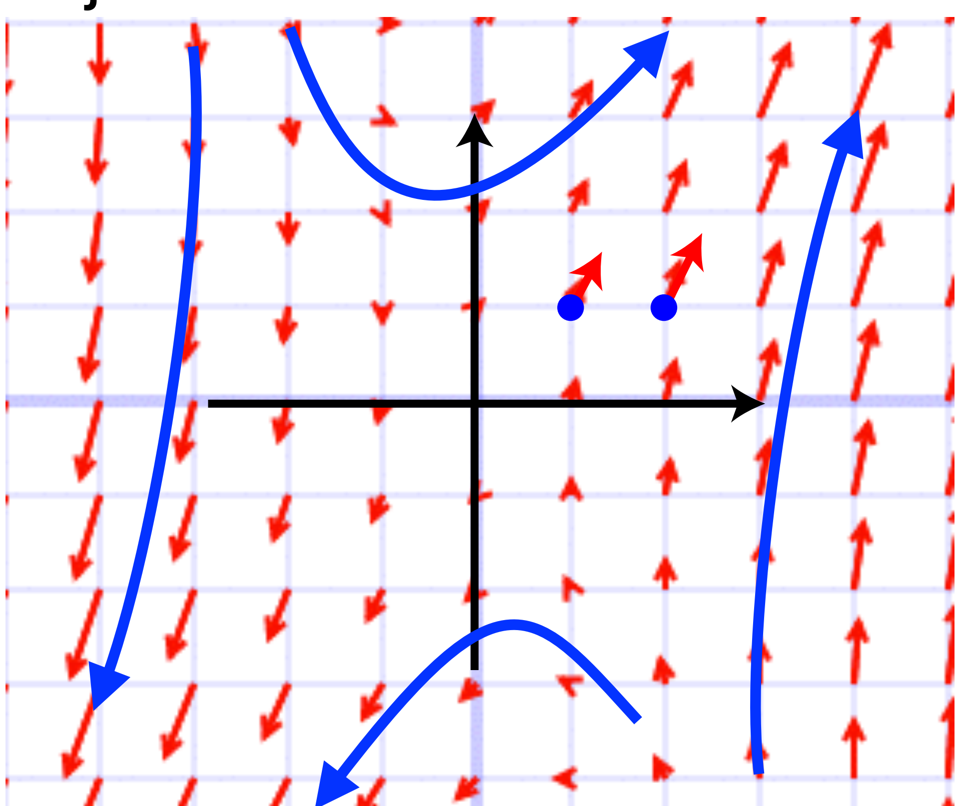

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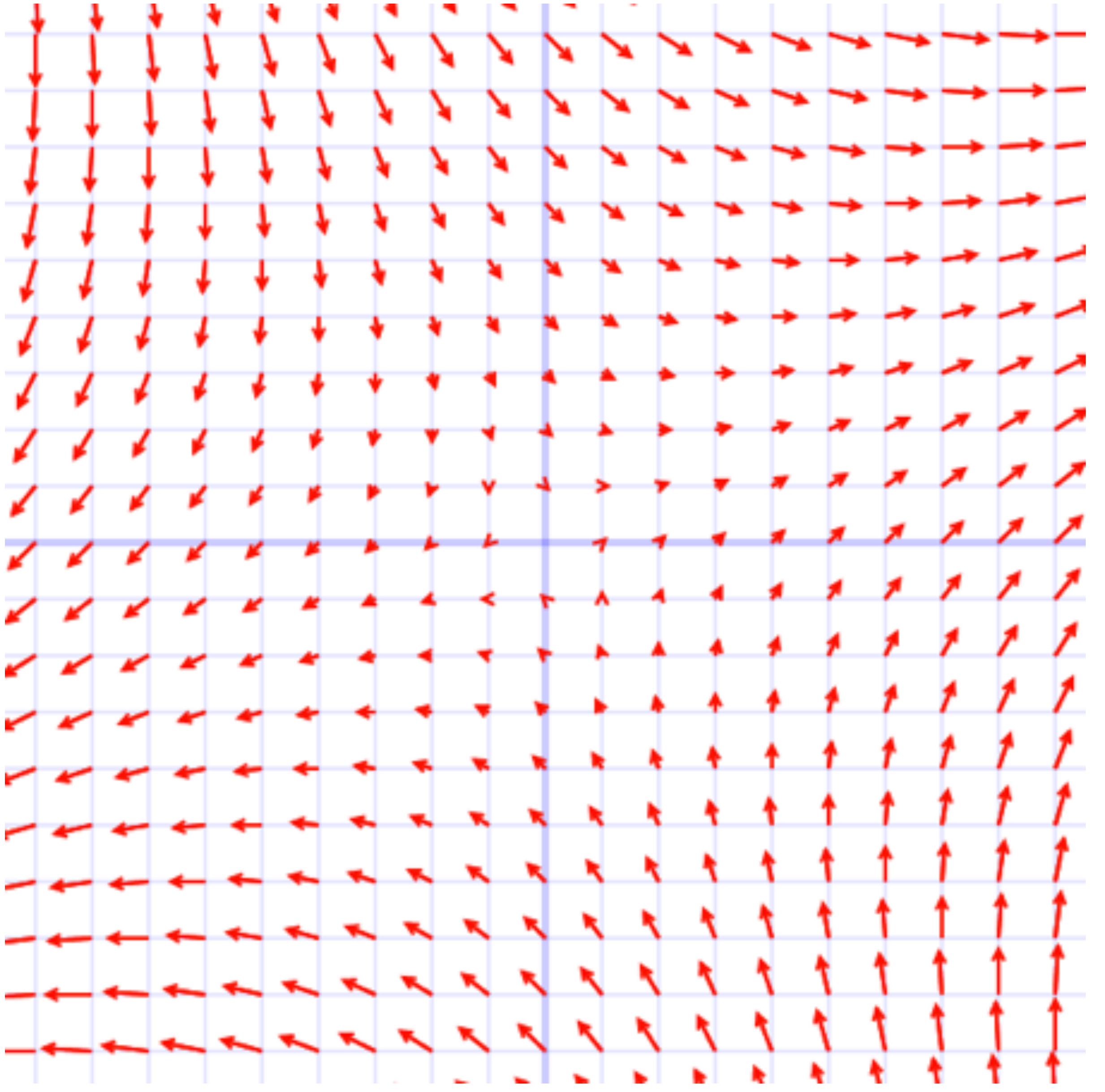

(E) Explain, please.

http://kevinmehall.net/p/equationexplorer/vectorfield.html

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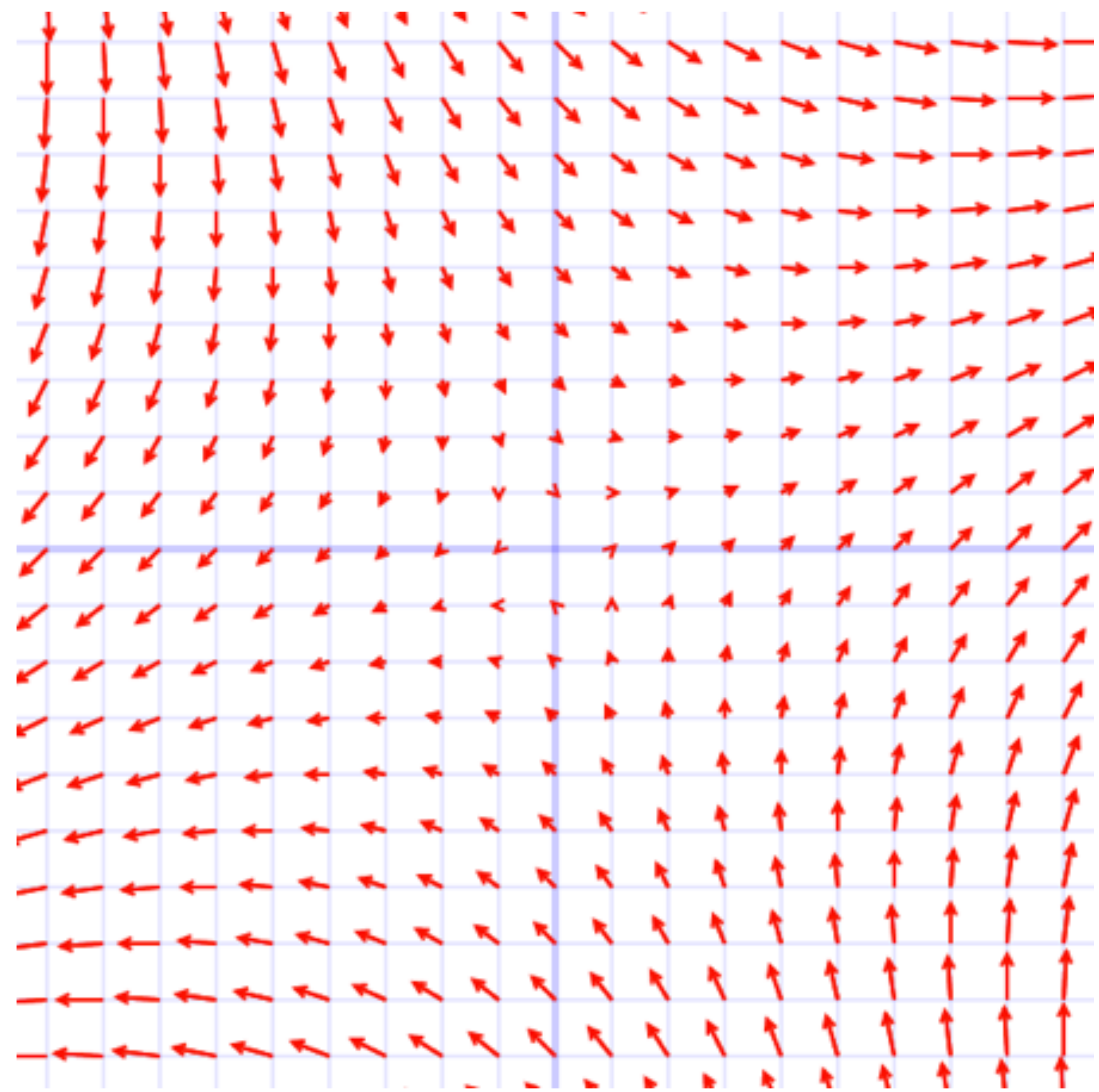

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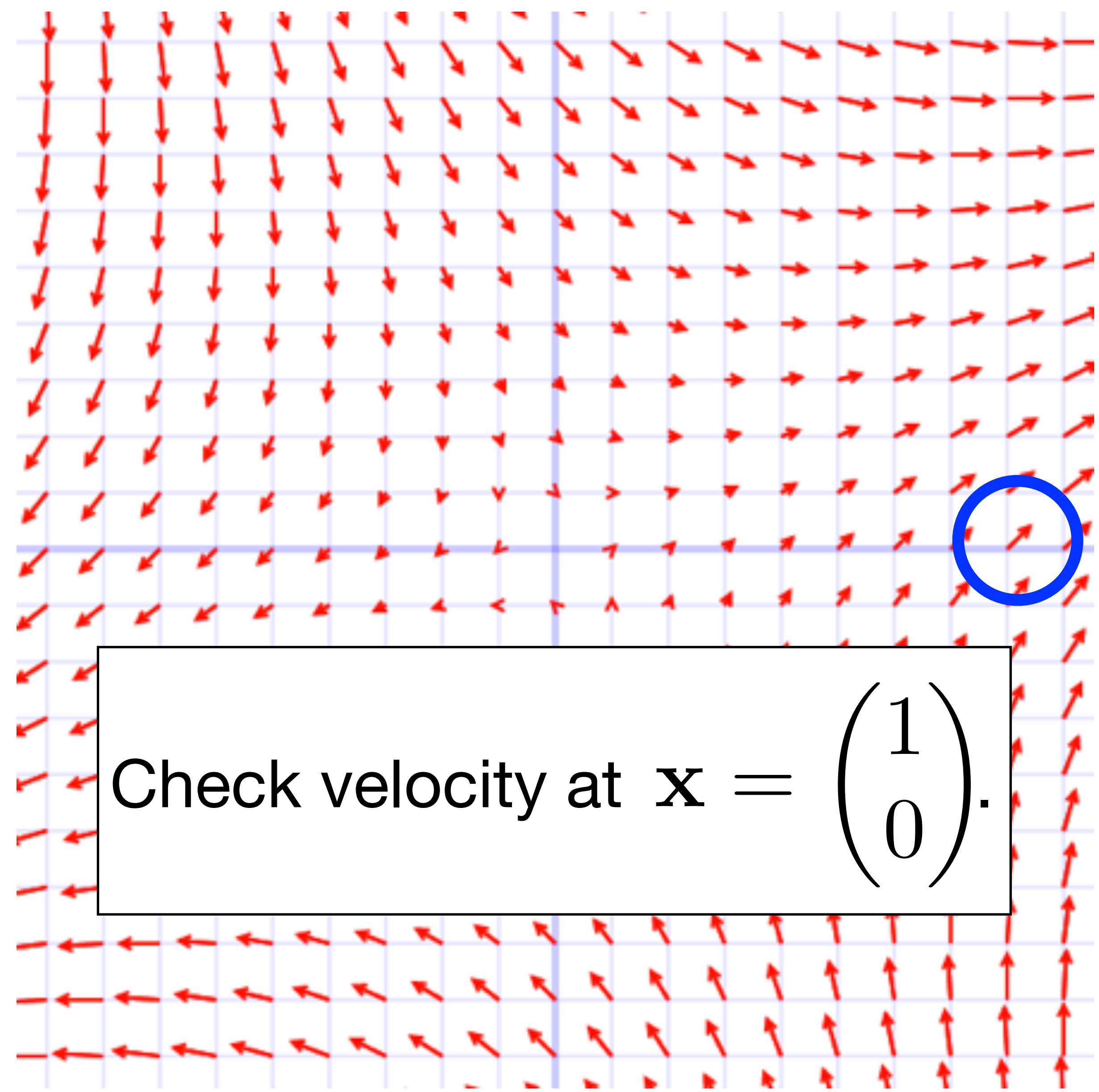

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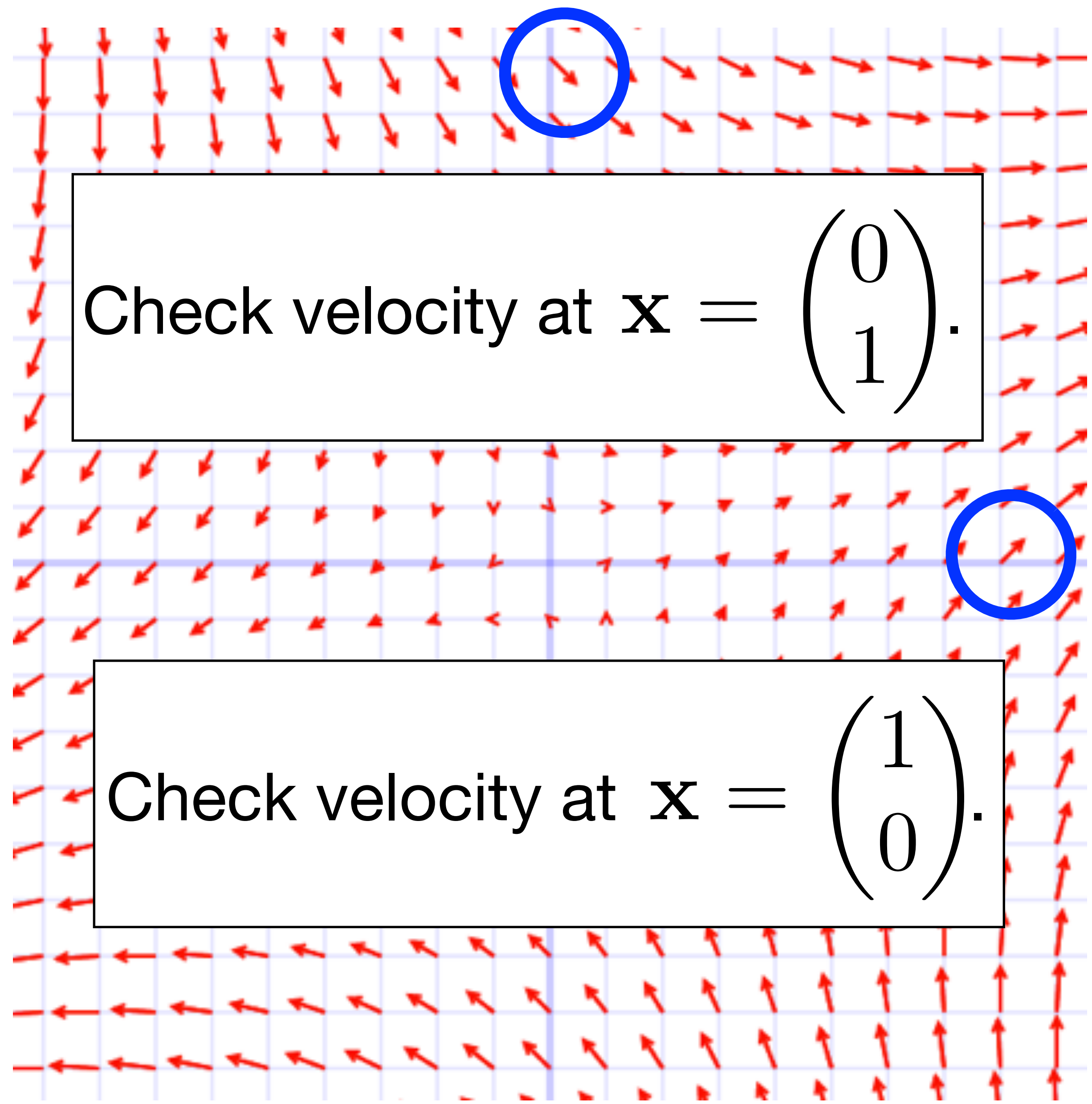

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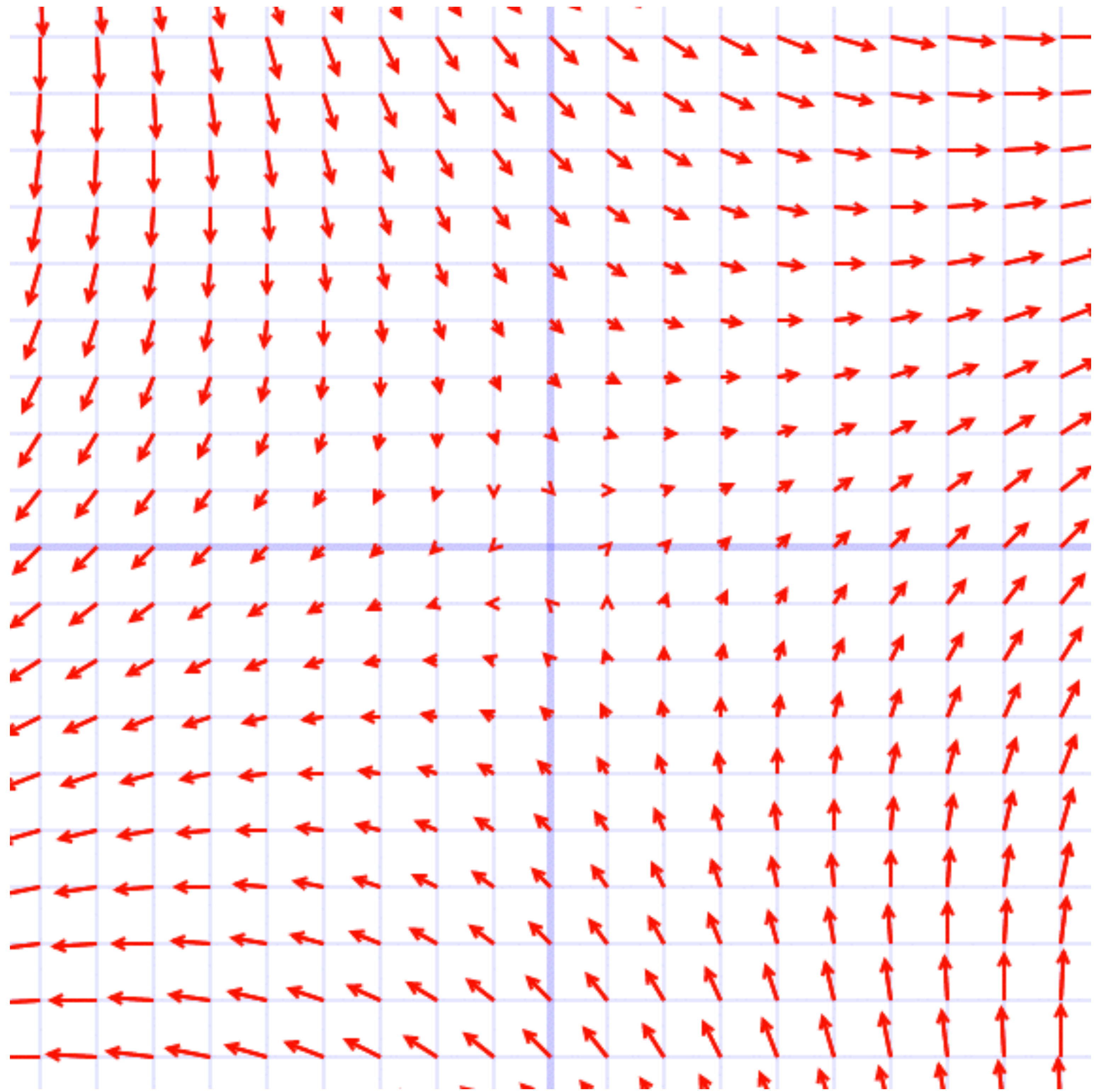
Introduction to systems of equations

- You should see two "special" directions.



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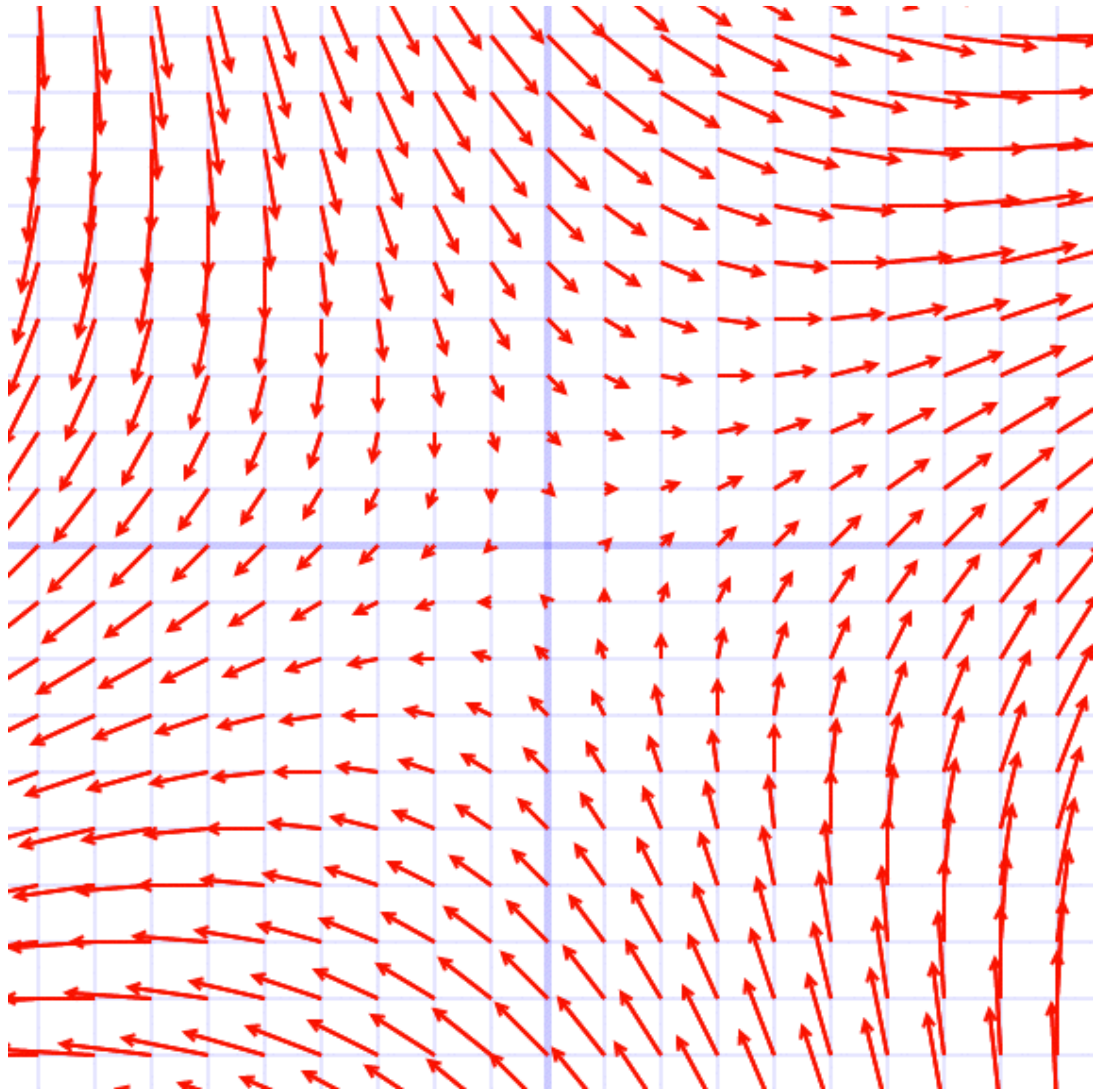

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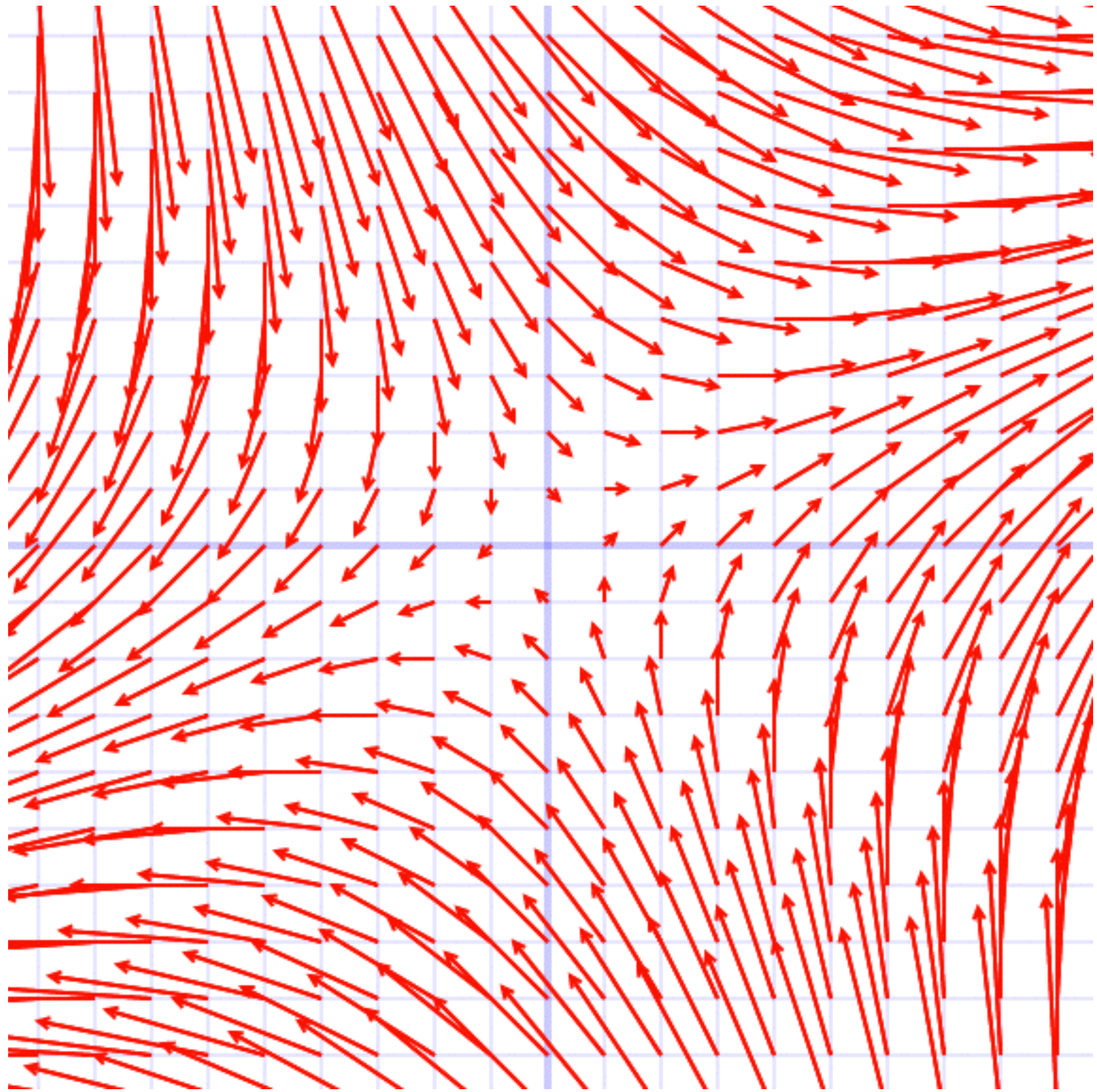

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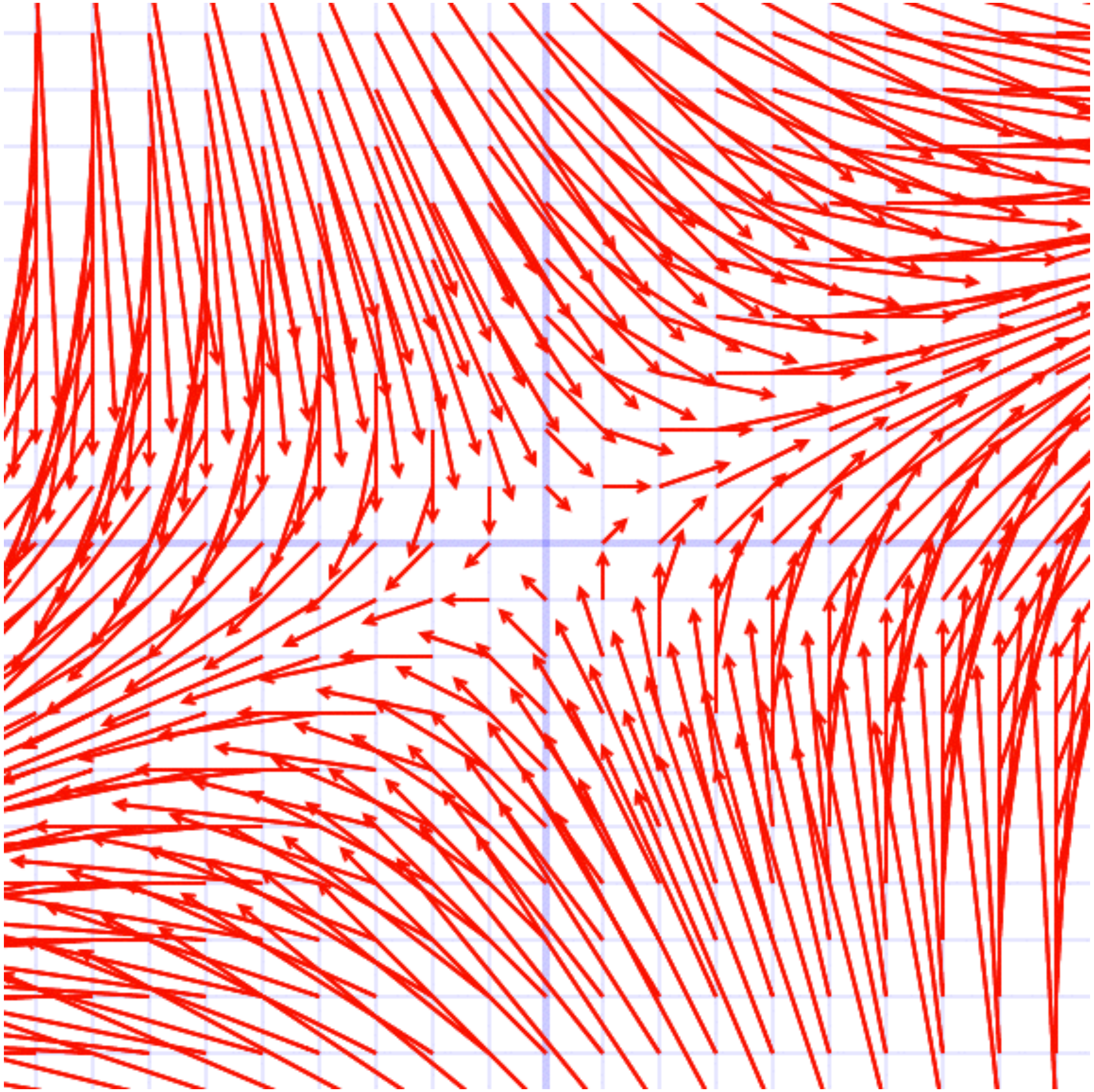

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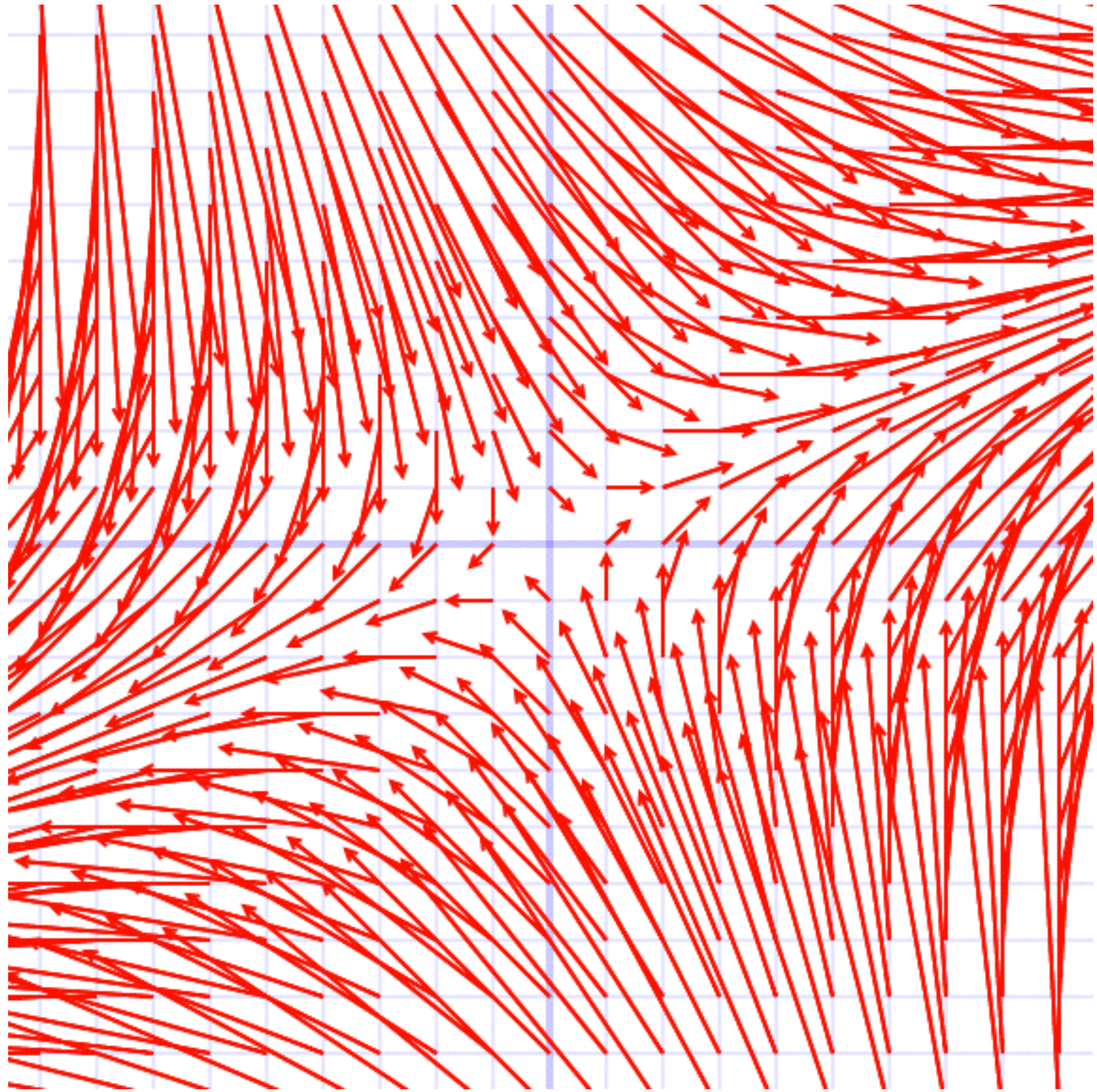

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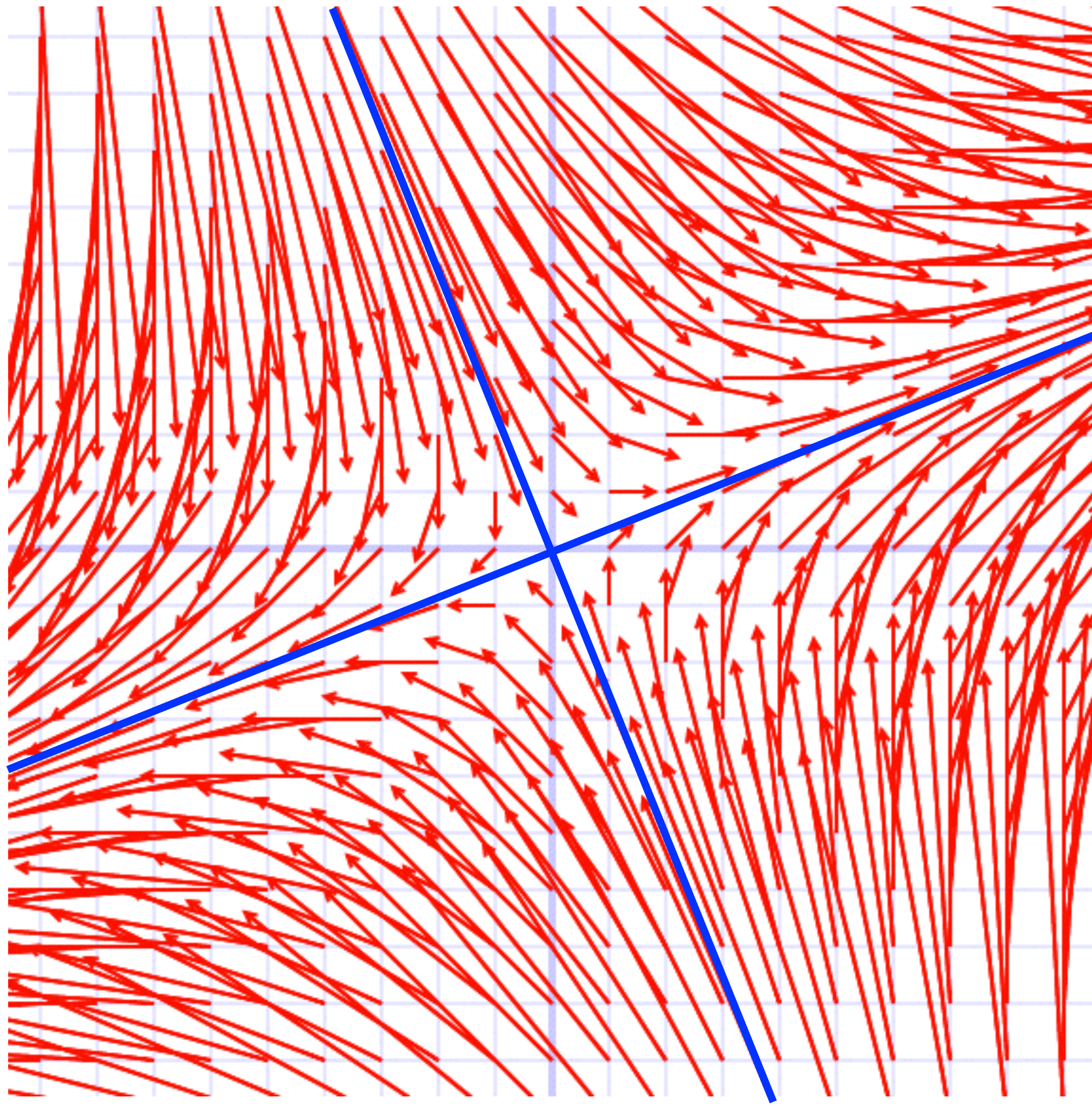

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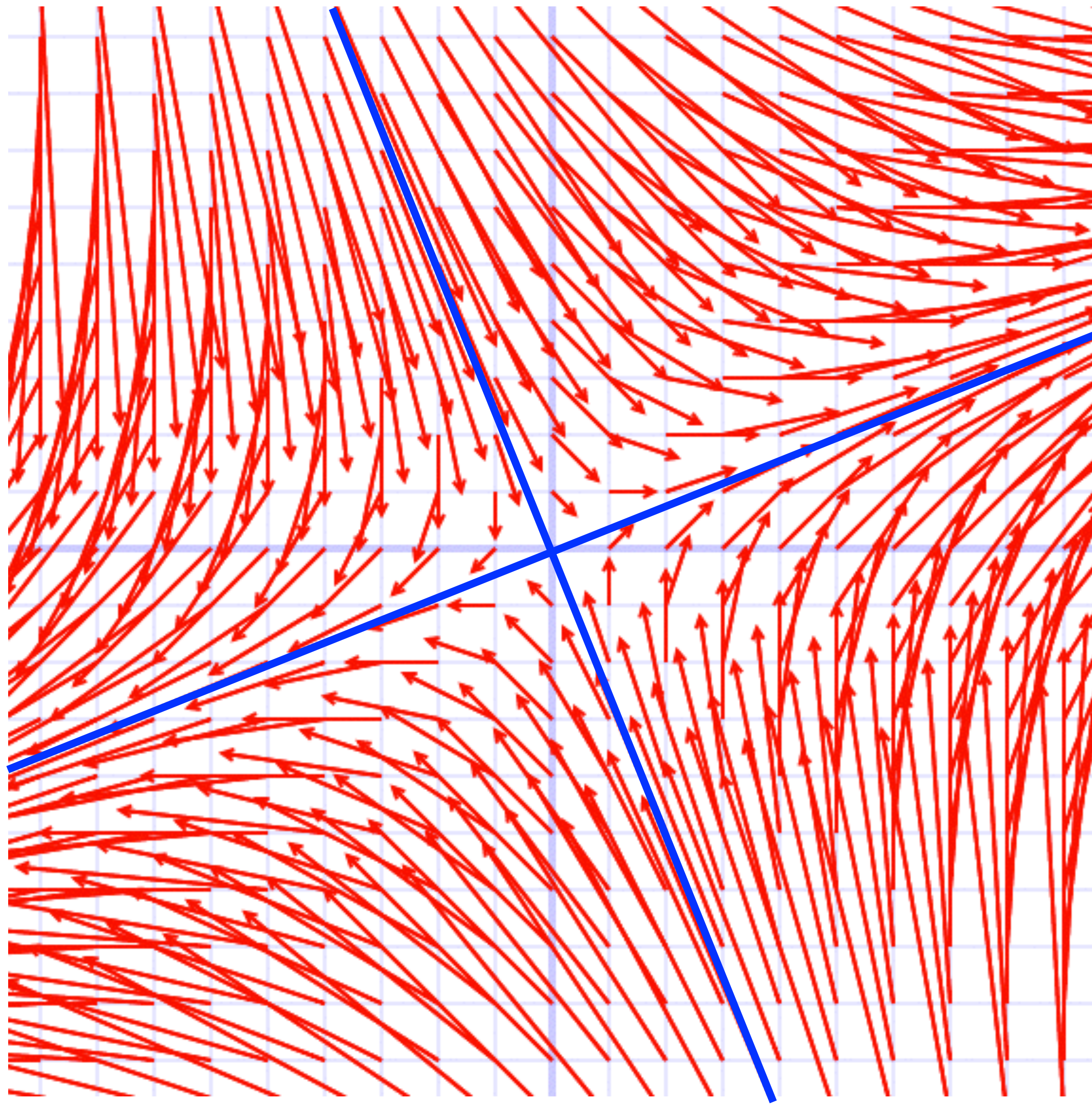

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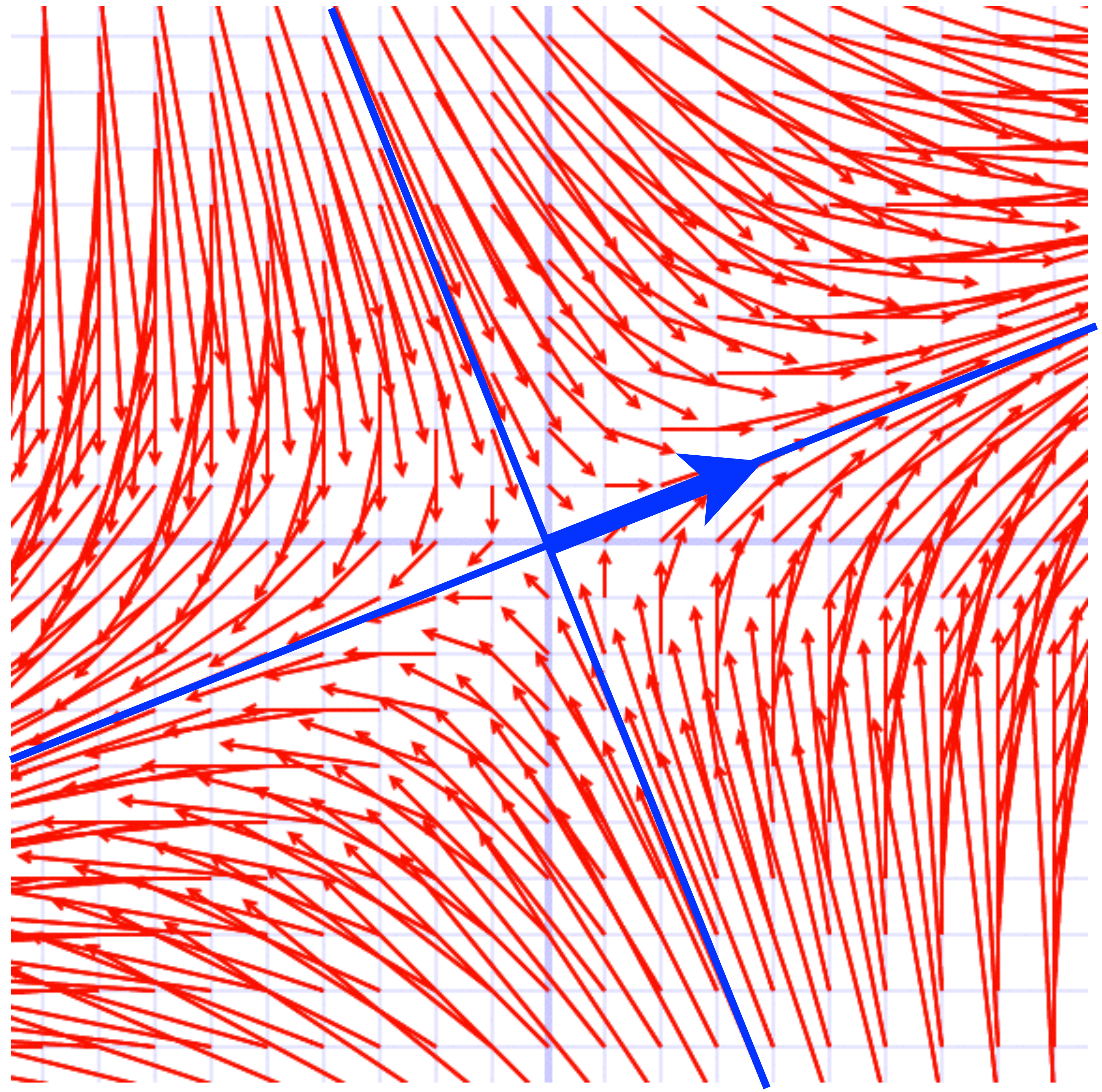

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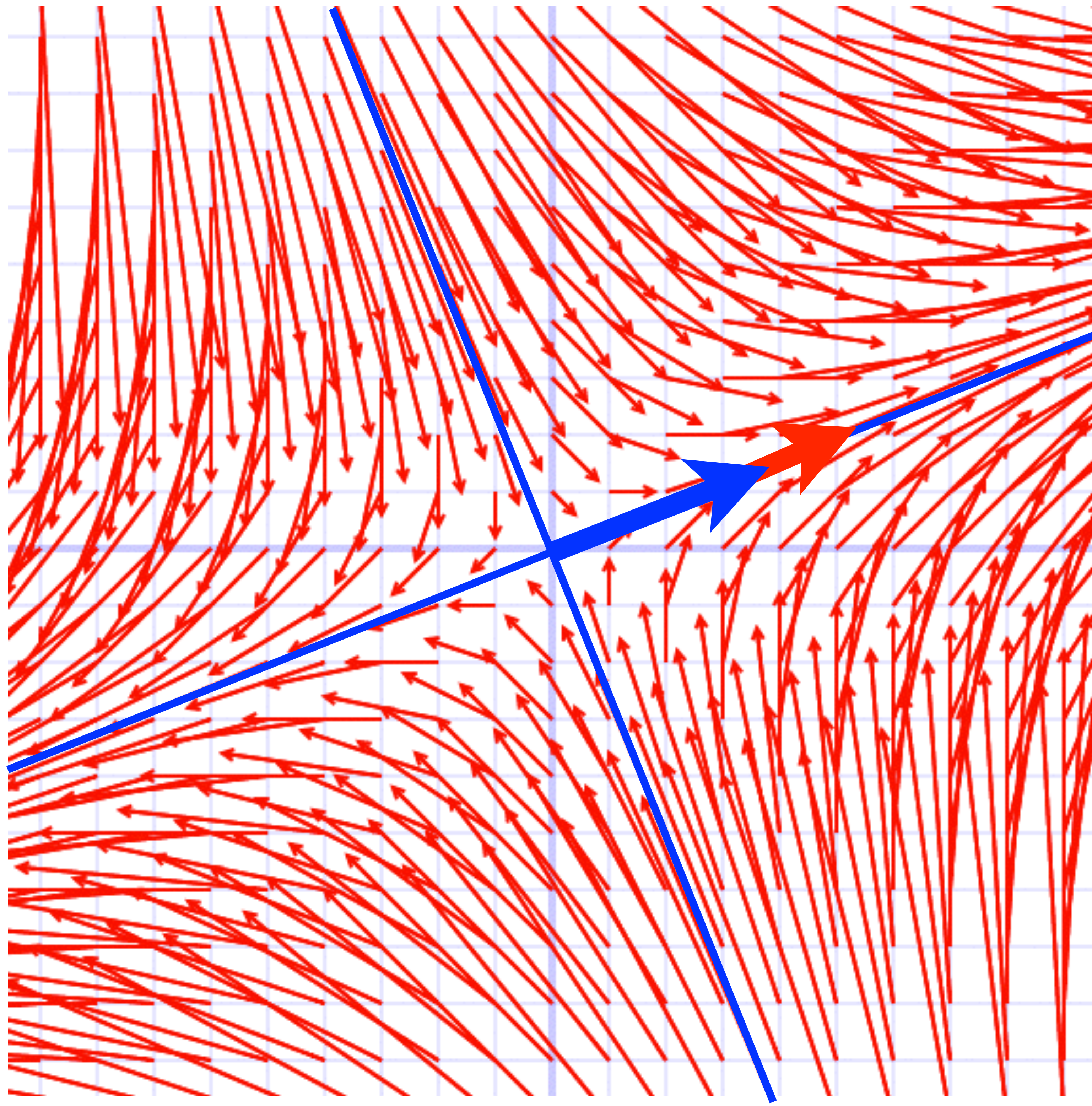

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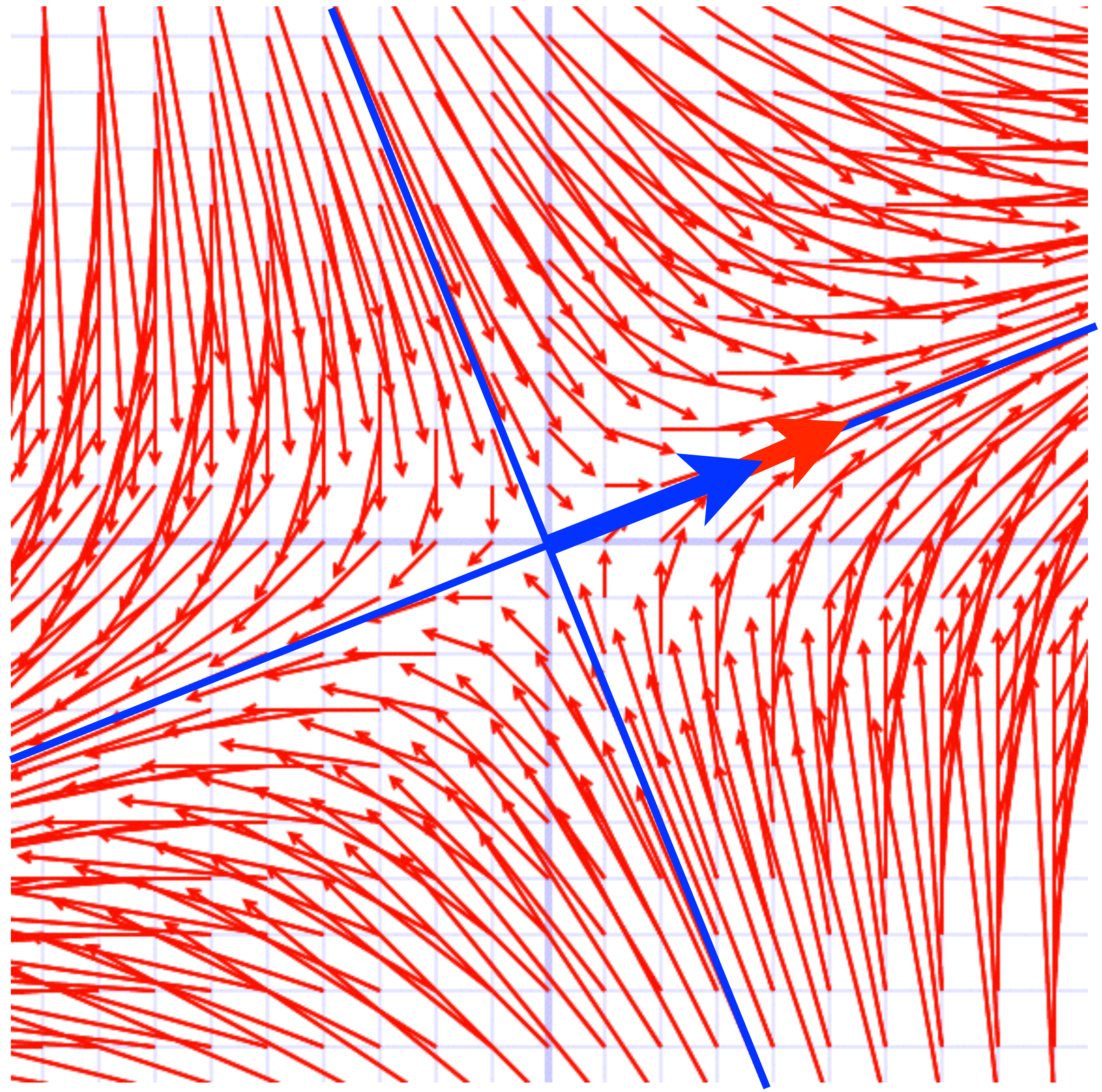
$$\lambda_1 = \sqrt{2}$$

$$\mathbf{v_1} = \begin{pmatrix} 1 \\ \sqrt{2} - 1 \end{pmatrix}$$



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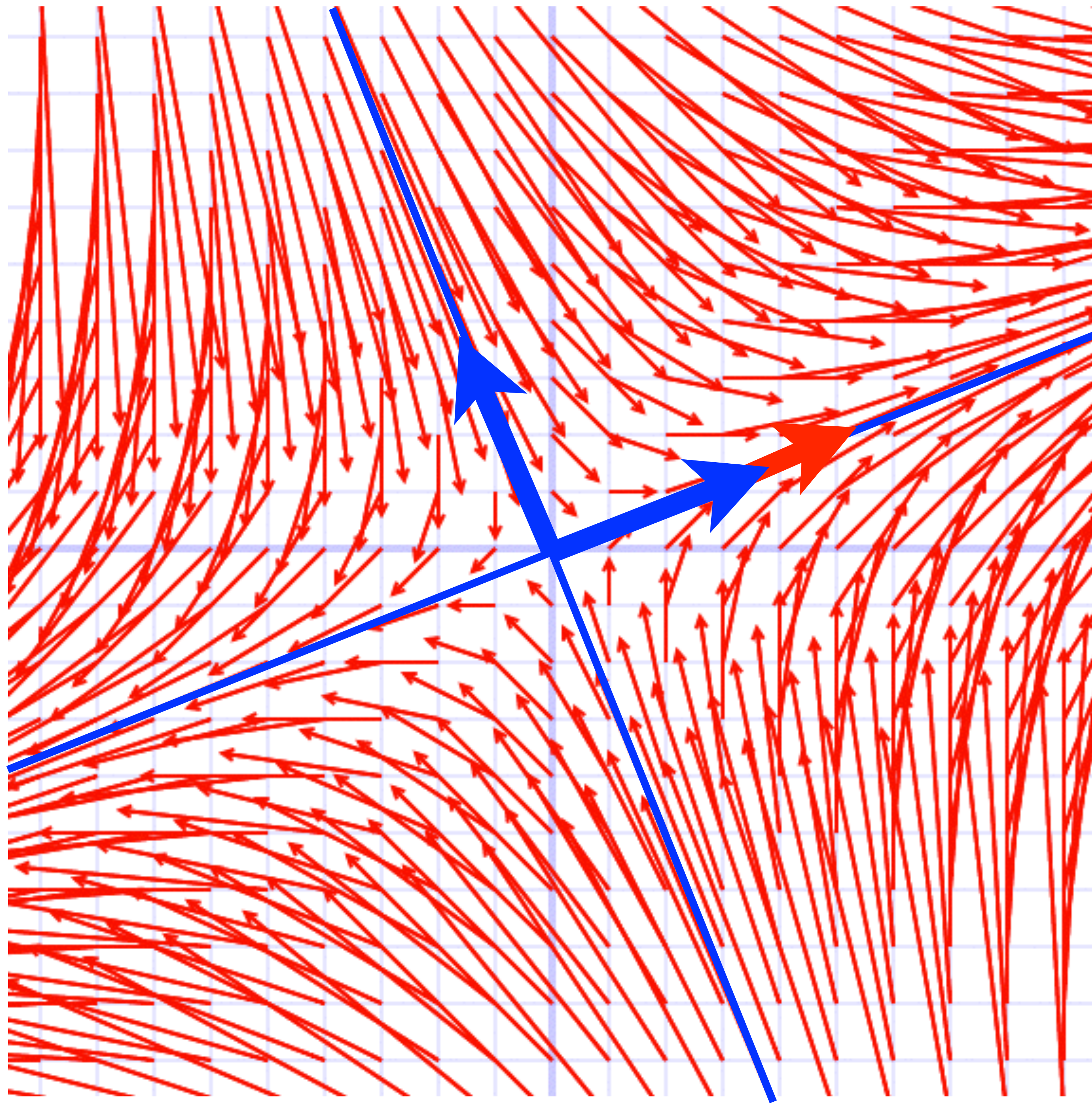

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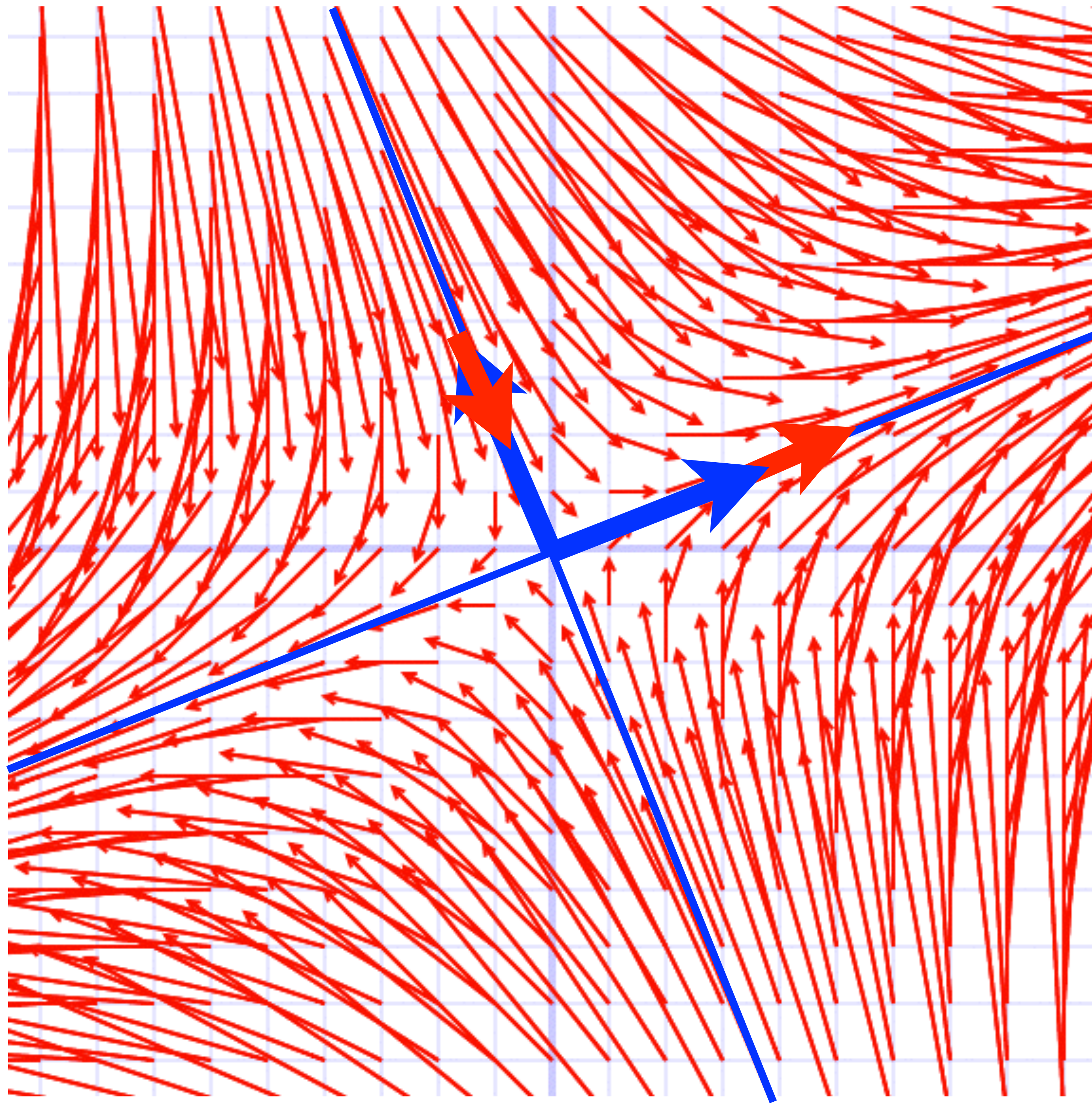

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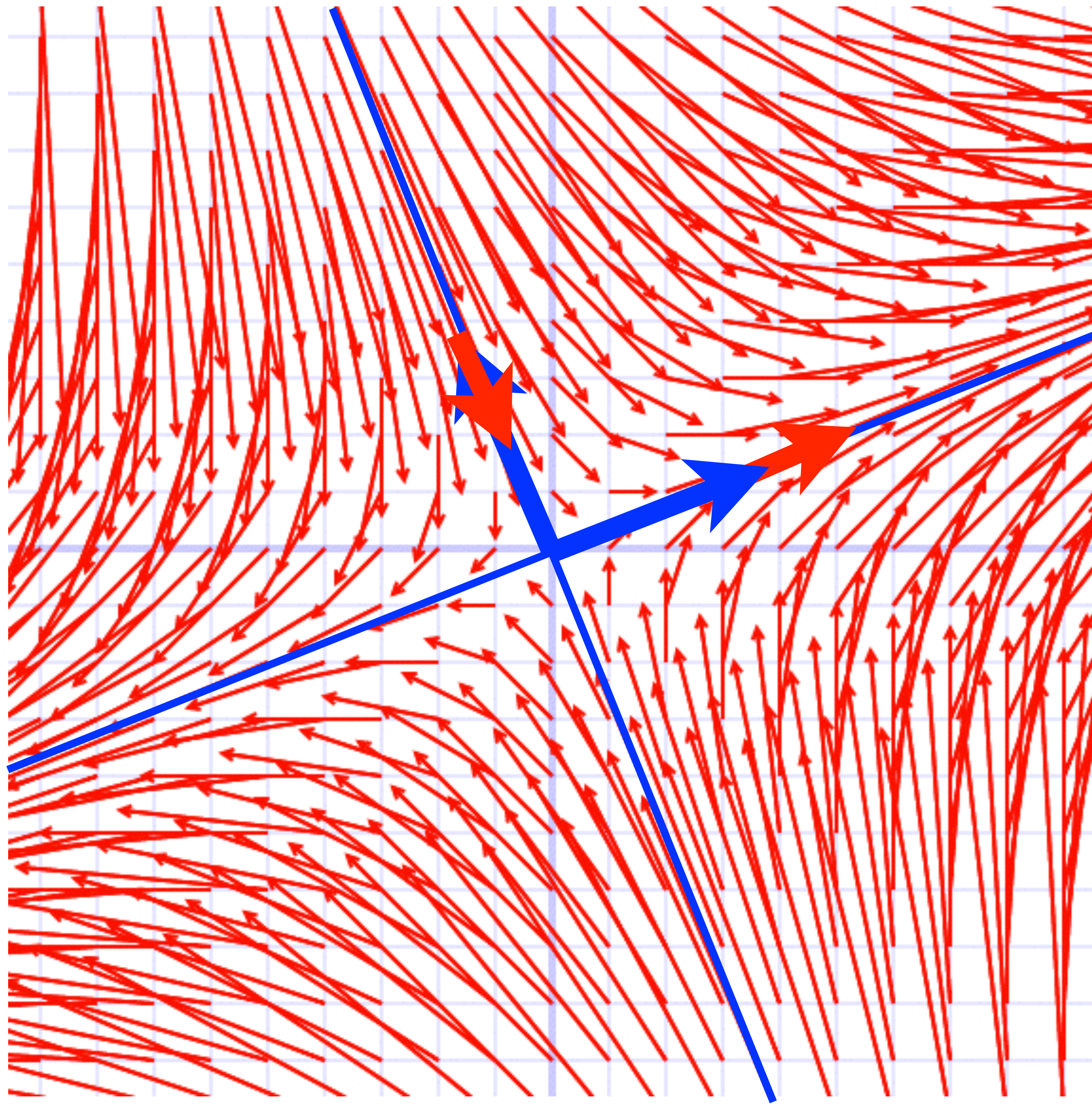

Matrix review (eigen-calculations)

- Find eigenvalues and eigenvectors of $A = \begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix}$.

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(B) -1 and 3

(C) 1 and 3

(D) -1 and -3

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$$\det(A - \lambda I) = 0$$

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(C) $\mathbf{v_1} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

(D) $\mathbf{v_1} = c\begin{pmatrix} 2 \\ 1 \end{pmatrix}$

(E) Explain, please.

Matrix review (eigen-calculations)

- Find eigenvalues and eigenvectors of $A = \begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix}$.
- Looking for values λ and vectors $\mathbf{v}$ for which $A\mathbf{v} = \lambda\mathbf{v}$.

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$$(A - \lambda I)\mathbf{v} = \mathbf{0}$$

$$\det(A - \lambda I) = 0$$

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$$(A + I)\mathbf{v_1} = \begin{pmatrix} 2 & 1 \\ 4 & 2 \end{pmatrix} \mathbf{v_1} = 0$$

$$\begin{pmatrix} 2 & 1 \\ 4 & 2 \end{pmatrix} \sim \begin{pmatrix} 2 & 1 \\ 0 & 0 \end{pmatrix}$$

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(and any scalar multiple of it)

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- How do we use eigenvalues and eigenvectors to construct a general solution?

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  - Other cases (not enough e-vectors or complex e-values) next class.