Today

- Midterm 1 - Jan 31 (one week away!)
- Finish up undetermined coefficients (on the midterm, on WW3)
- Physics applications - mass springs (not on midterm, on WW4)
- Undamped, over/under/critically damped oscillations (maybe Thurs)

Method of undetermined coefficients

- Example. Find the general solution to $y'' + 2y' = e^{2t} + t^3$.
  - What is the form of the particular solution?

(A) $y_p(t) = Ae^{2t} + Bt^3 + Ct^2 + Dt$

(B) $y_p(t) = Ae^{2t} + Bt^3 + Ct^2 + Dt + E$

☆ (C) $y_p(t) = Ae^{2t} + (Bt^4 + Ct^3 + Dt^2 + Et)$
$y_p(t) = Ae^{2t} + t(Bt^3 + Ct^2 + Dt + E)$

(D) $y_p(t) = Ae^{2t} + Be^{-2t} + Ct^3 + Dt^2 + Et + F$

(E) Don't know / still thinking.

For each wrong answer, for what DE is it the correct form?

Method of undetermined coefficients

- Example. Find the general solution to $y'' - 4y = t^3 e^{2t}$.
  - What is the form of the particular solution?

(A) $y_p(t) = (At^3 + Bt^2 + Ct + D)e^{2t}$

(B) $y_p(t) = (At^3 + Bt^2 + Ct)e^{2t}$

(C) $y_p(t) = (At^3 + Bt^2 + Ct)e^{2t} + (Dt^3 + Et^2 + Ft)e^{-2t}$

☆ (D) $y_p(t) = (At^4 + Bt^3 + Ct^2 + Dt)e^{2t}$
$y_p(t) = t(At^3 + Bt^2 + Ct + D)e^{2t}$

(E) Don't know / still thinking.

Method of undetermined coefficients

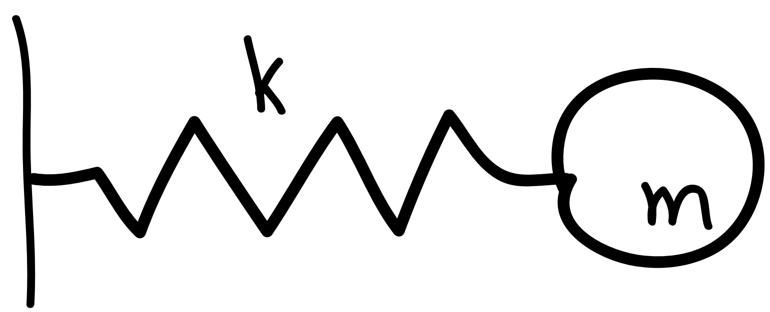
- Summary - finding a particular solution to L[y] = g(t).
  - Include all functions that are part of the g(t) family (e.g. cos **and** sin)
  - If part of the g(t) family is a solution to the homogeneous (h-)problem, use t x (g(t) family).
  - If t x (part of the g(t) family), is a solution to the h-problem, use t^2 x (g(t) family). etc.
  - For sums, group terms into families and include terms for each. You can even find a y_p for each family and add them up.
  - Works for products of functions - be sure to include the whole family!
  - Never include a solution to the h-problem as it won't survive L[ ]. Just make sure you aren't missing another term somewhere.

Method of undetermined coefficients

- Do lots of these problems and the trends will become clear.
- Try different y_ps and see what goes wrong - this will help you see what must happen when things go right.
- Two crucial facts to remember
    - If you try a form and you can make LHS=RHS with some choice for the coefficients then you're done.
    - If you can't, your guess is most likely missing a term(s).

Applications - vibrations

Mass-spring systems



$$E = \frac{1}{2}k(x-x_0)^2$$

$$F = -\frac{dE}{dx} = -k(x-x_0)$$

$$ma = F$$

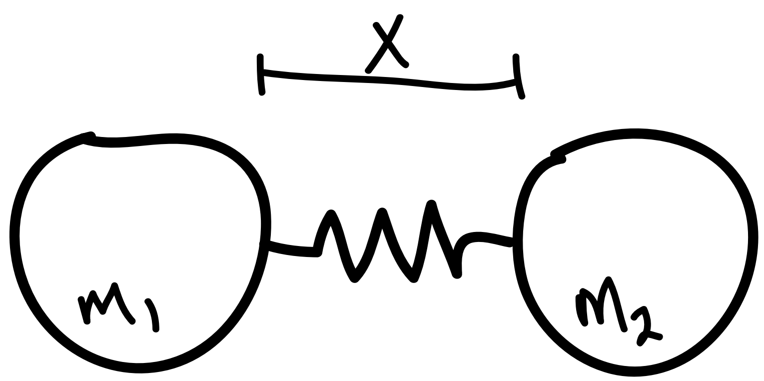
$$ma = -k(x-x_0)$$

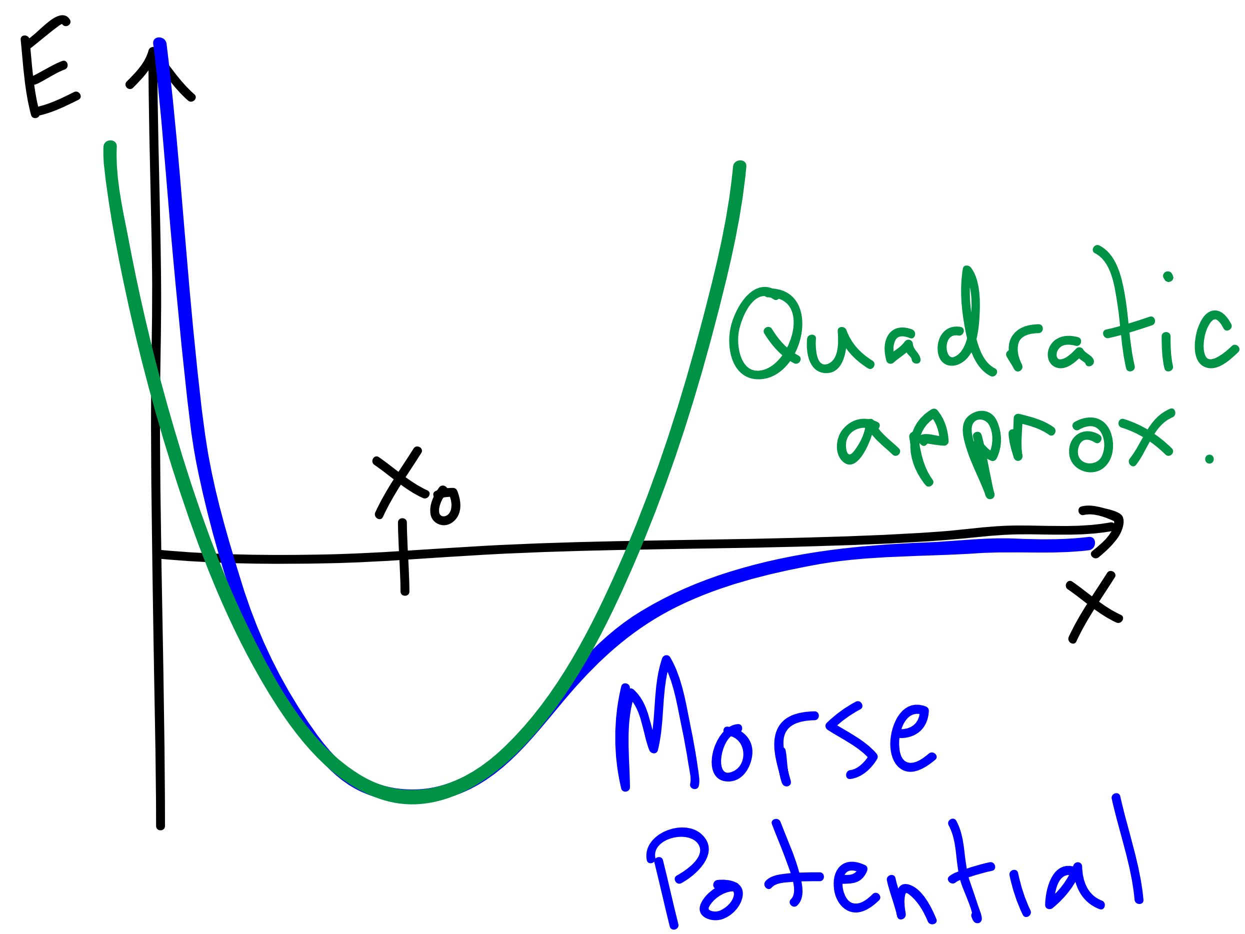
$$mx'' = -k(x-x_0)$$

$$mx'' + kx = kx_0$$

Applications - vibrations

Molecular bonds

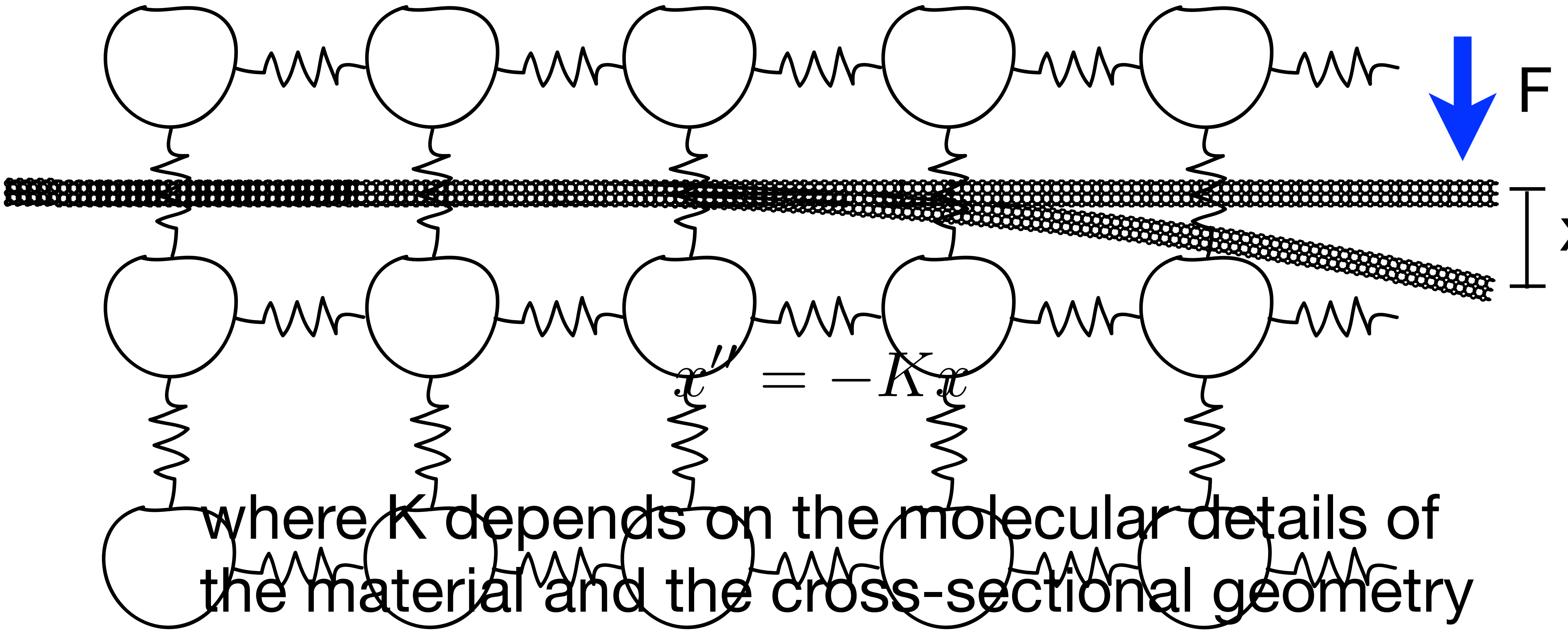




Applications - vibrations

Solid mechanics

e.g. tuning fork, bridges, buildings



$$x'' = -Kx$$

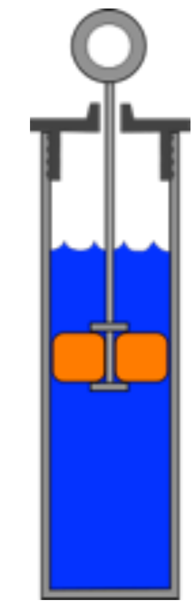
where K depends on the molecular details of the material and the cross-sectional geometry of the rod.

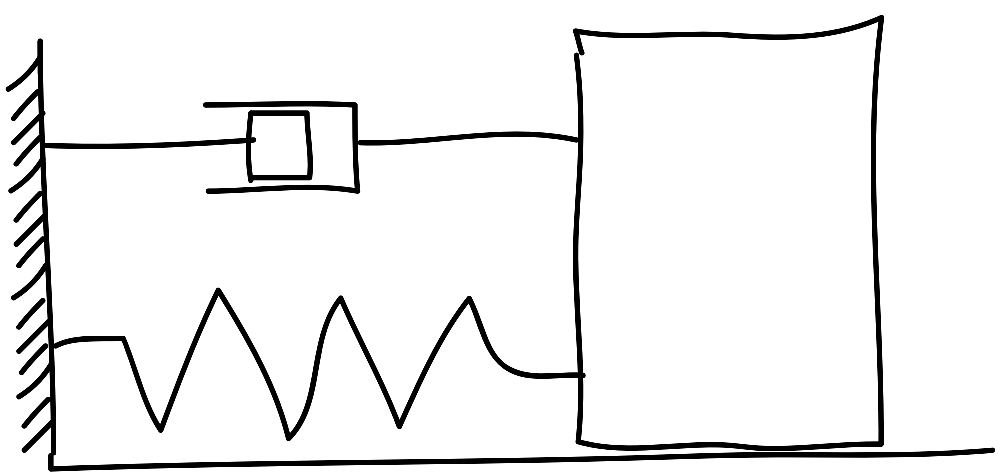
Applications - vibrations

- So far, no x’ term so no exponential decay in the solutions.

- Dashpot - mechanical element that adds friction.

  - sometimes an abstraction that accounts for energy loss.





Kelvin-Voigt model

$$ma = -k(x - x_0) - \gamma v$$

$$mx'' = -k(x - x_0) - \gamma x'$$

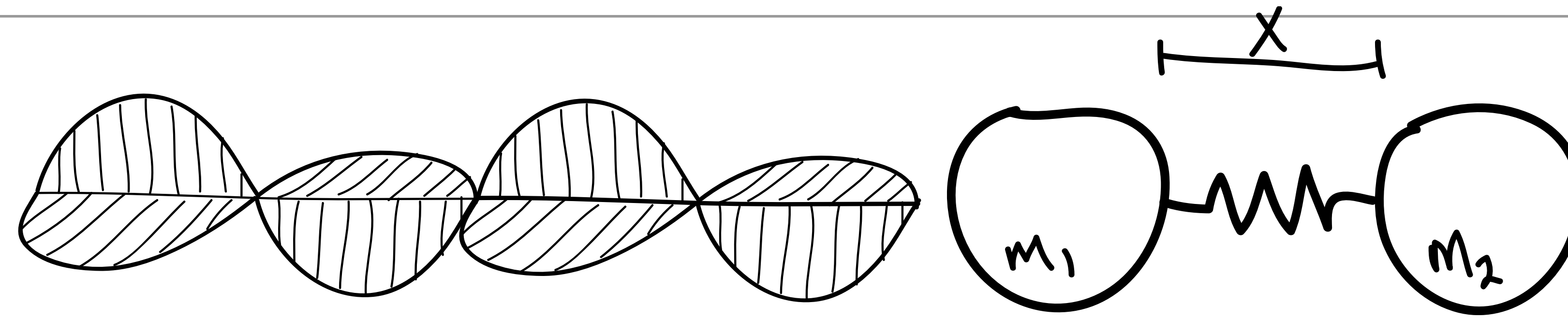
$$mx'' + \gamma x' + kx = kx_0$$

$$y = x - x_0$$

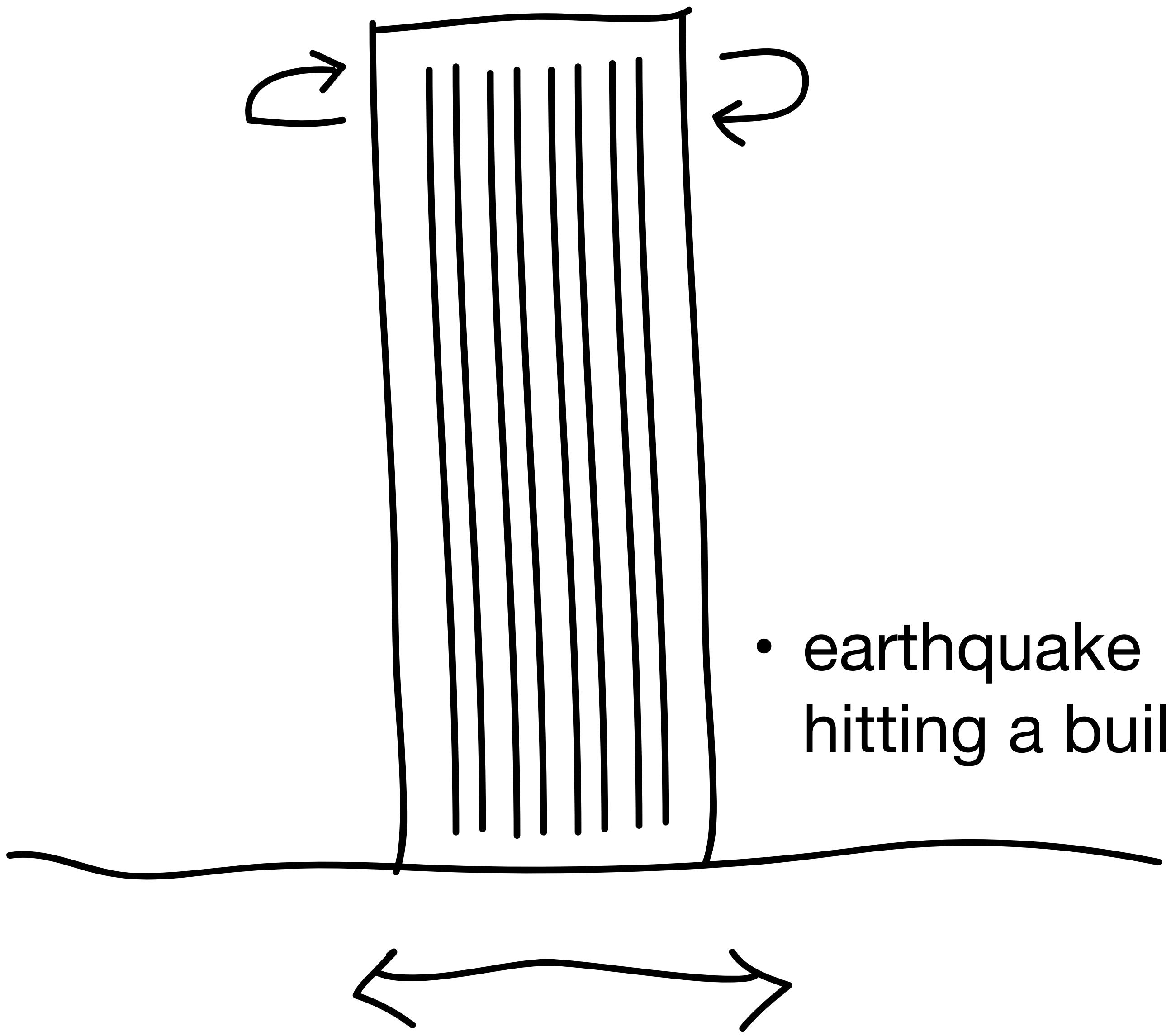
$$my'' + \gamma y' + ky = 0$$

shock absorber

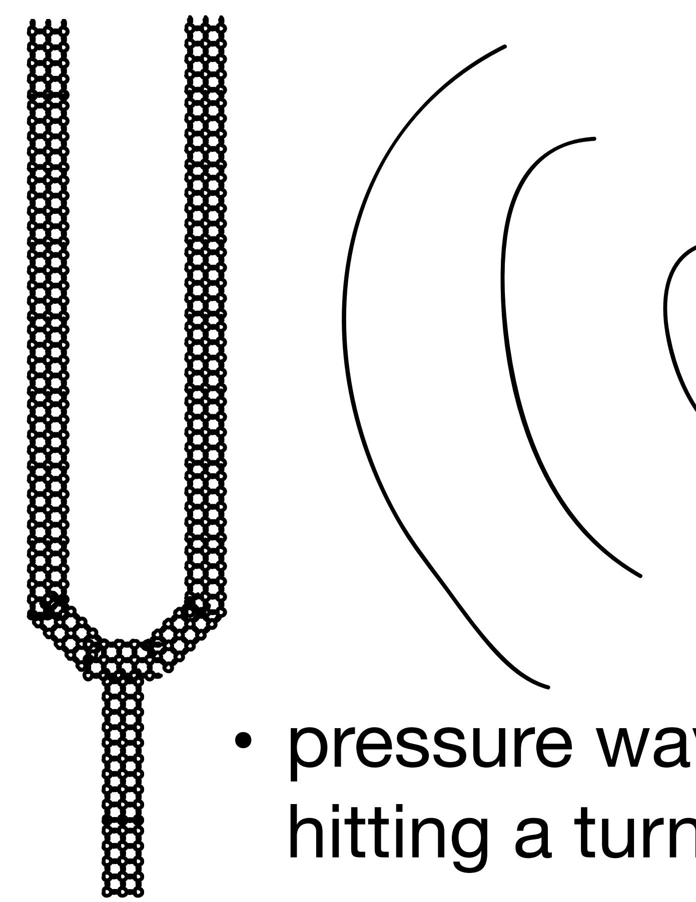
Applications - forced vibrations



- light hitting a molecular bond



- earthquake hitting a building



- pressure waves (sound) hitting a turning fork.