Today

- Midterm 1 - Jan 31 (one week away!)
- Finish up undetermined coefficients (on the midterm, on WW3)
- Physics applications - mass springs (not on midterm, on WW4)
- Undamped, over/under/critically damped oscillations (maybe Thurs)

Method of undetermined coefficients

- Example. Find the general solution to $y'' + 2y' = e^{2t} + t^3$.
  - What is the form of the particular solution?

(A) $y_p(t) = Ae^{2t} + Bt^3 + Ct^2 + Dt$

(B) $y_p(t) = Ae^{2t} + Bt^3 + Ct^2 + Dt + E$

(C) $y_p(t) = Ae^{2t} + (Bt^4 + Ct^3 + Dt^2 + Et)$

(D) $y_p(t) = Ae^{2t} + Be^{-2t} + Ct^3 + Dt^2 + Et + F$

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For each wrong answer, for what DE is it the correct form?

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    - Never include a solution to the h-problem as it won't survive L[ ]. Just make sure you aren't missing another term somewhere.

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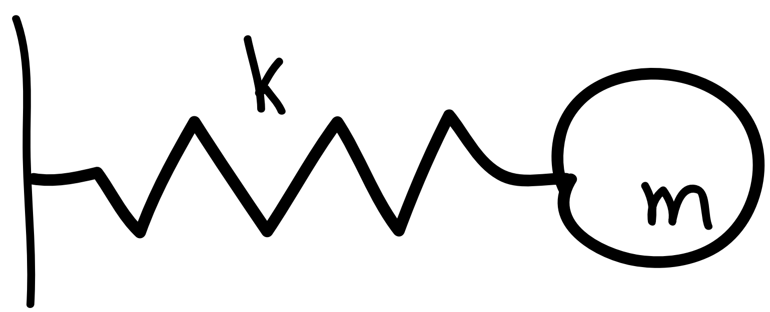
    - If you try a form and you can make LHS=RHS with some choice for the coefficients then you're done.
    - If you can't, your guess is most likely missing a term(s).

Applications - vibrations

Mass-spring systems



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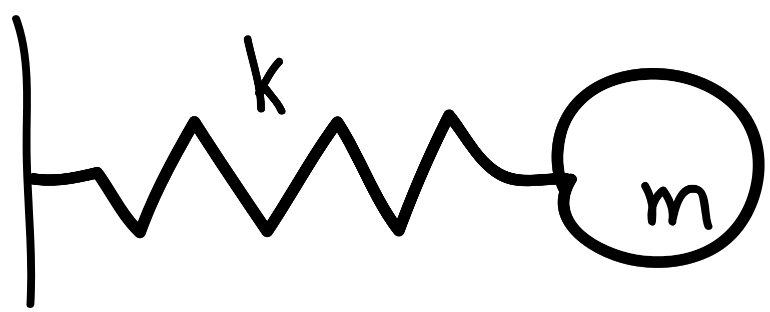

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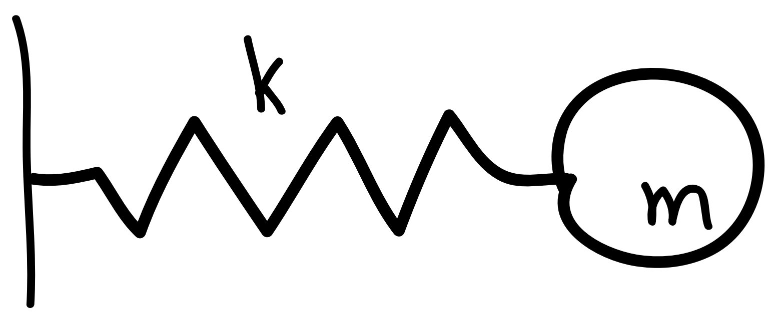

$$E = \frac{1}{2}k(x - x_0)^2$$

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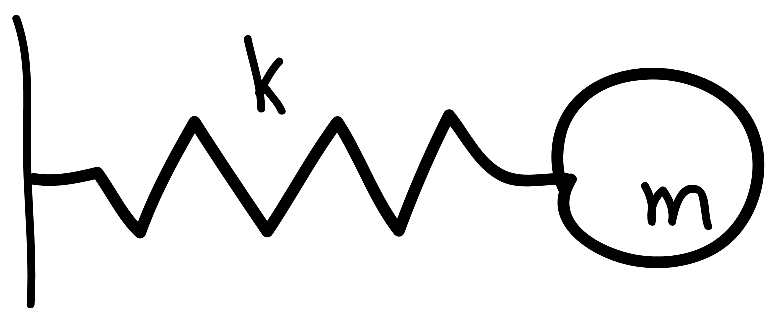

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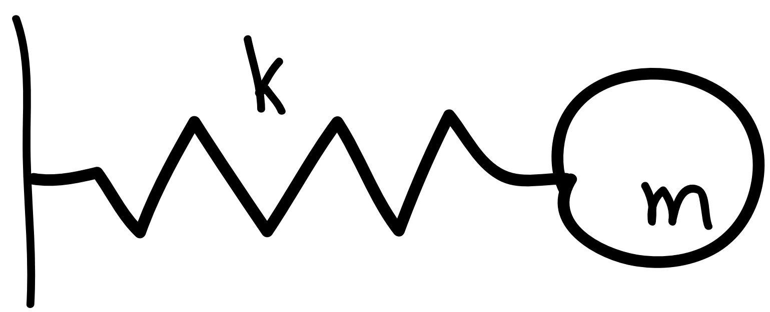

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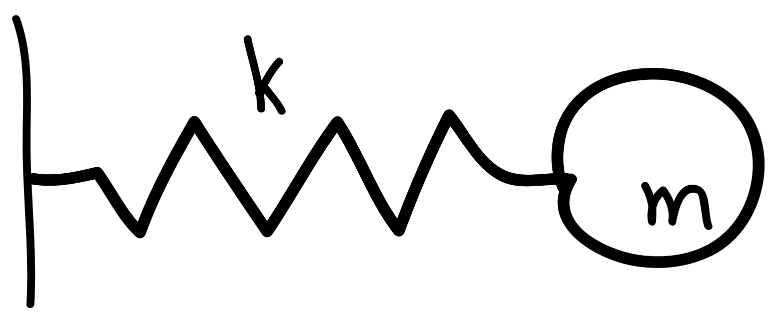

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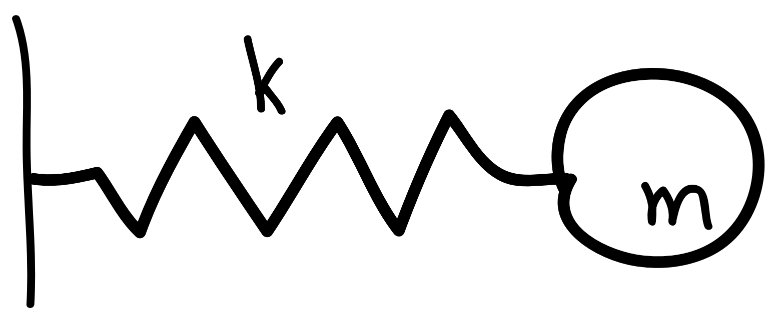

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Molecular bonds

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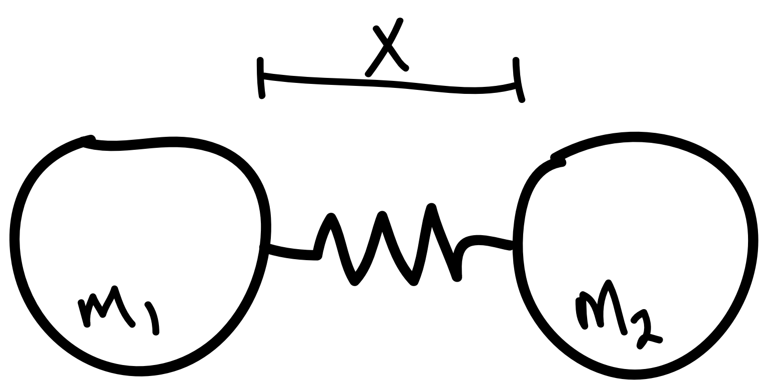

Applications - vibrations

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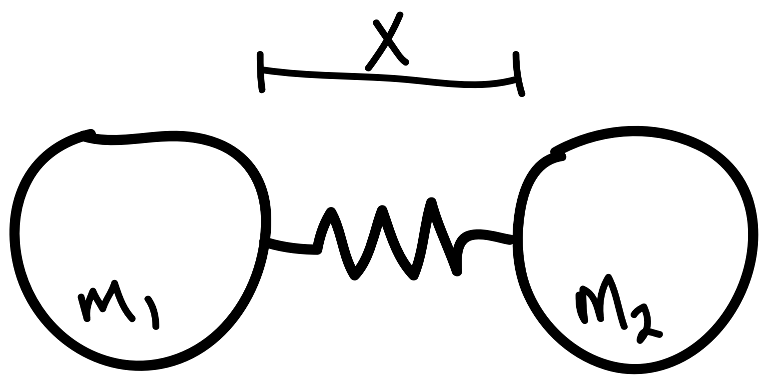

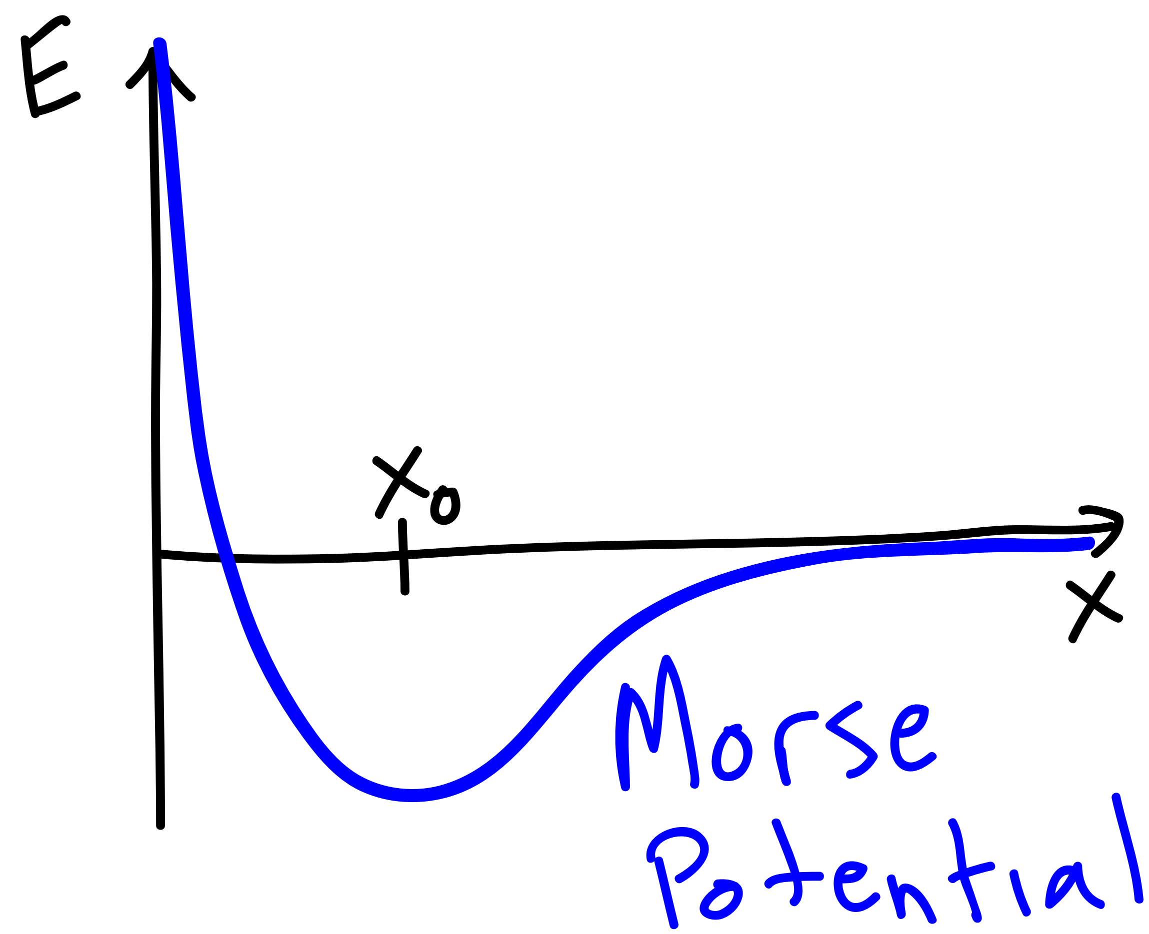

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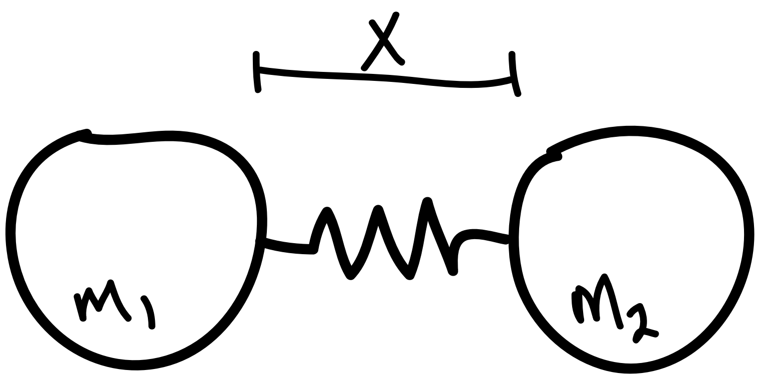
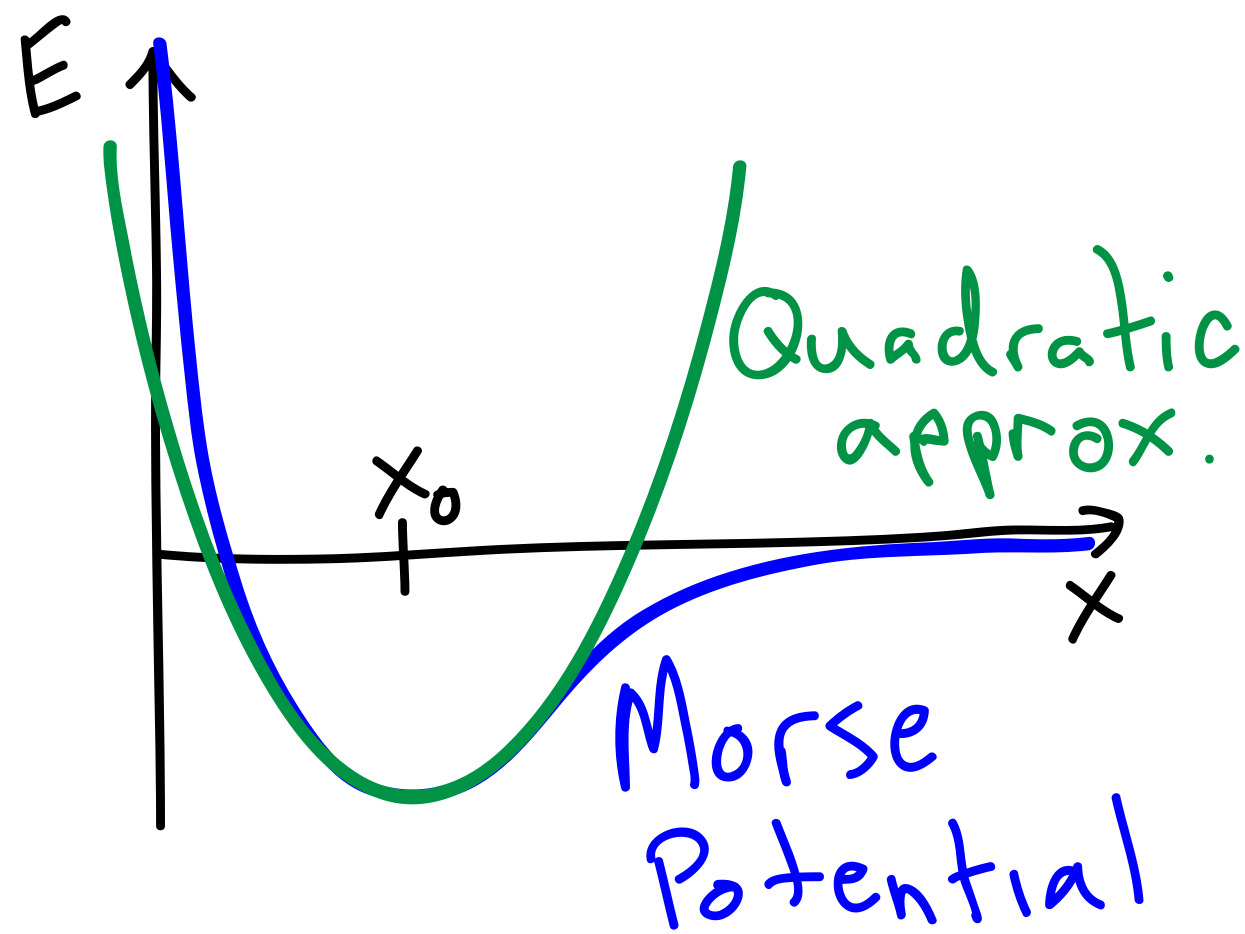
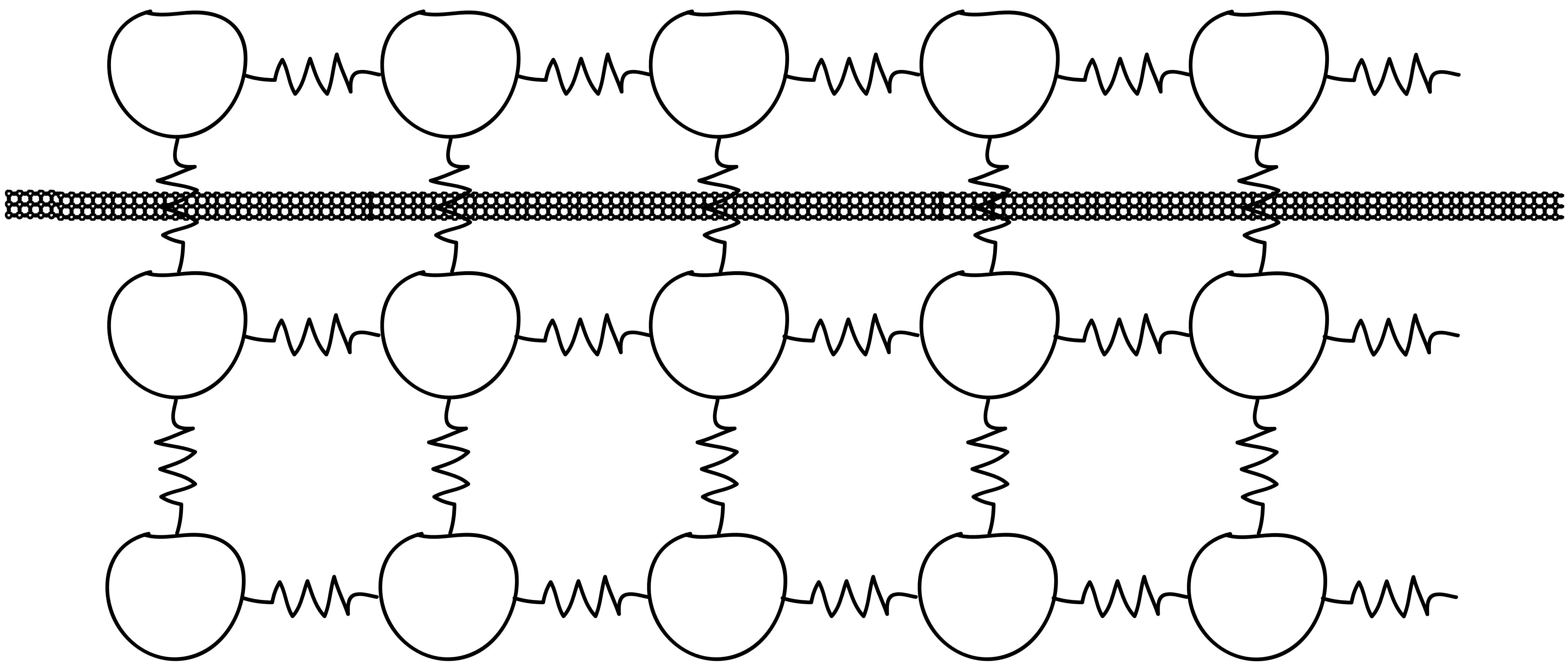




Applications - vibrations

Solid mechanics

e.g. tuning fork, bridges, buildings



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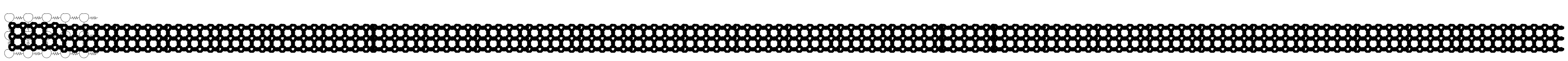

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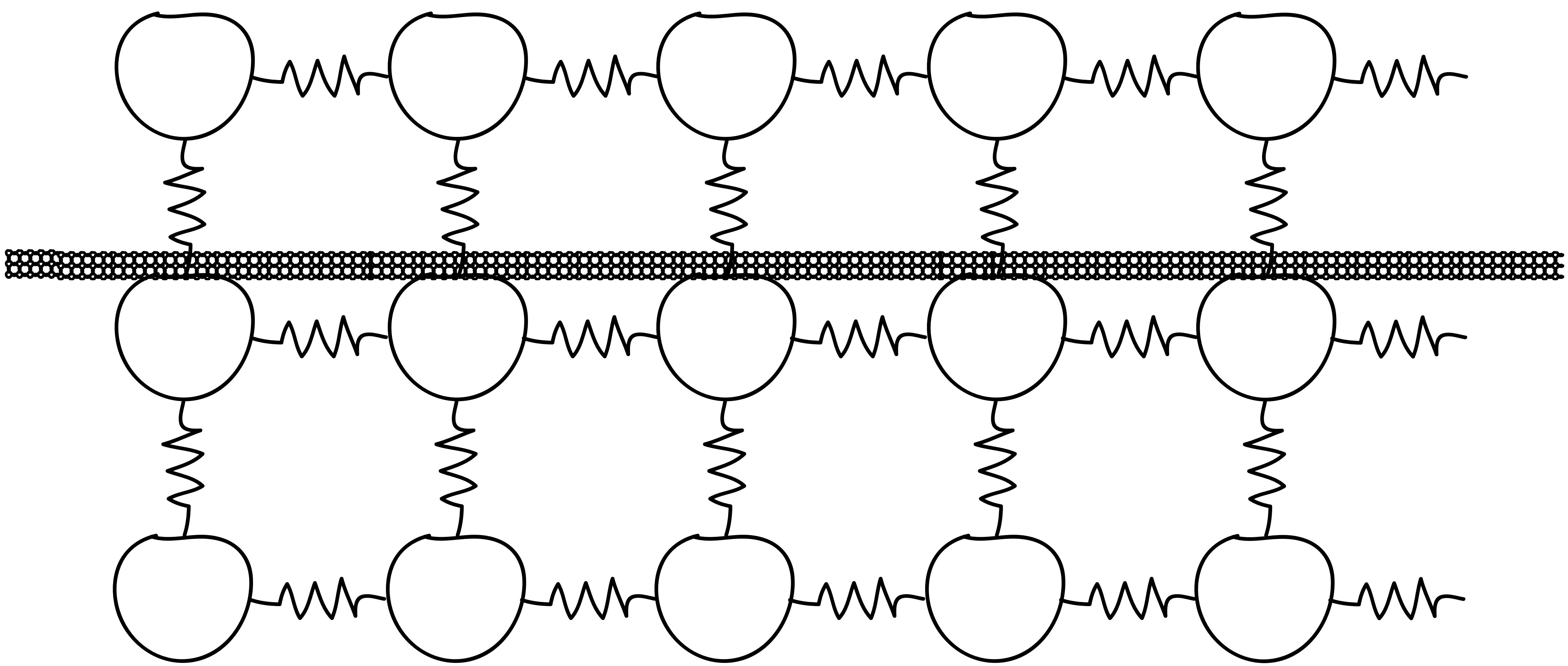

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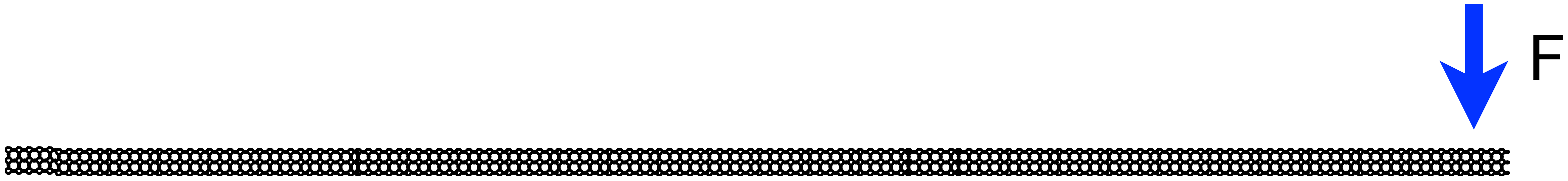

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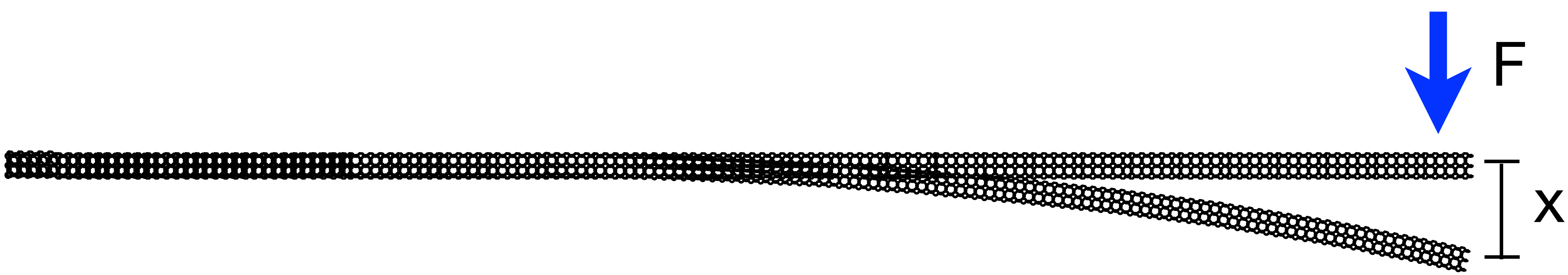

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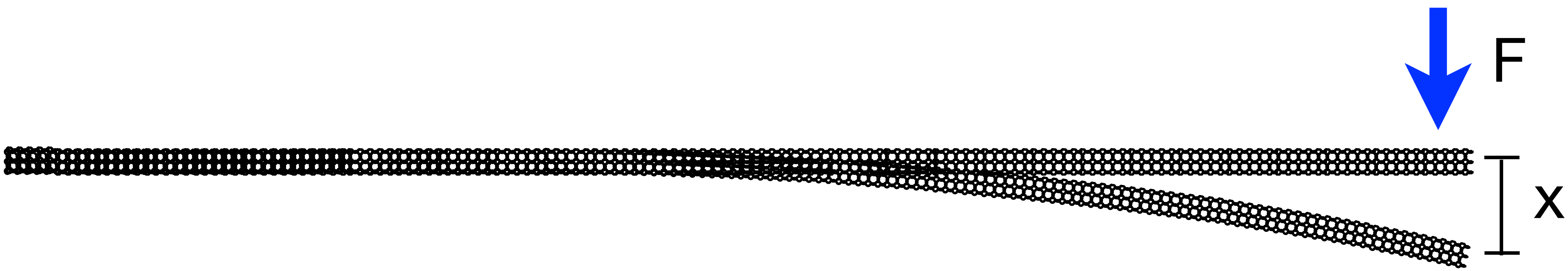

$$x'' = -Kx$$

where K depends on the molecular details of the material and the cross-sectional geometry of the rod.

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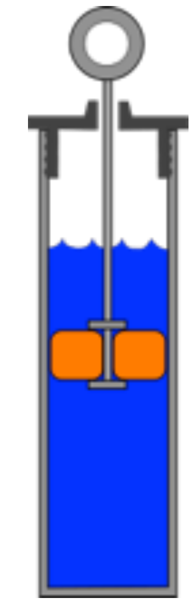

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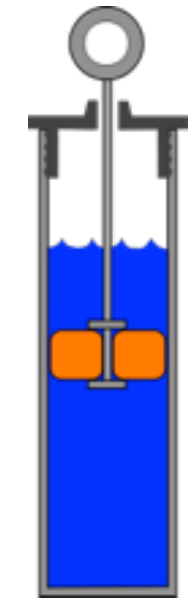

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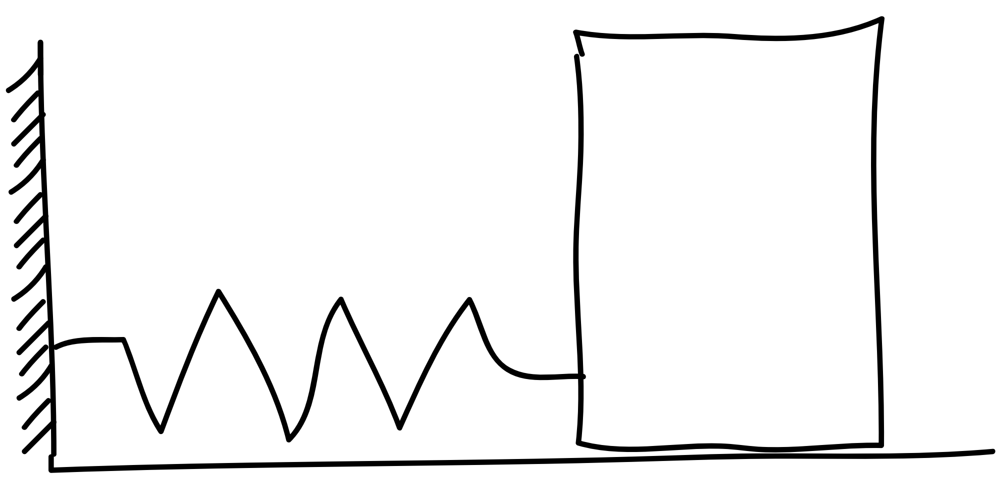

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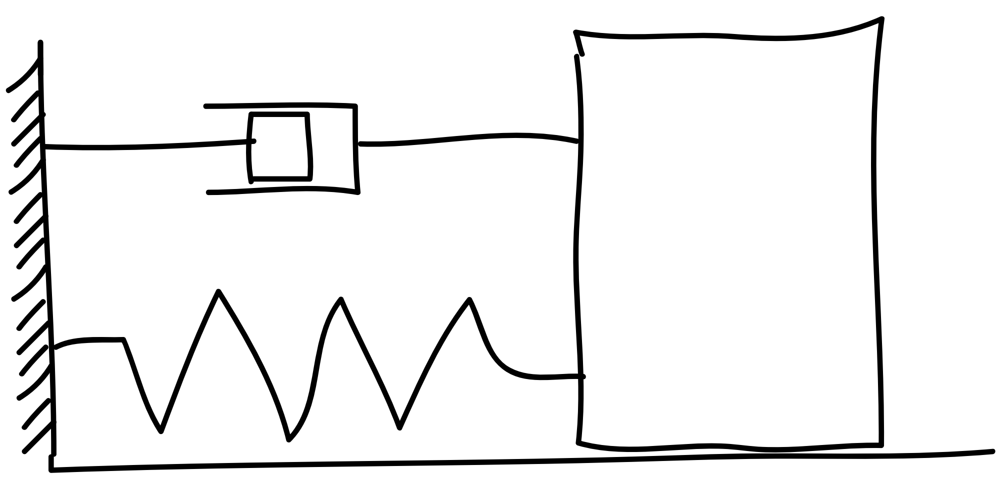

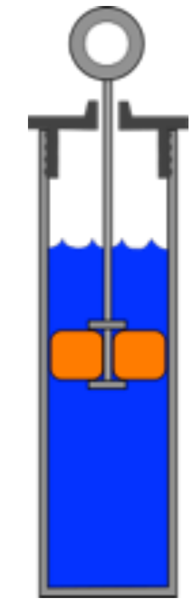

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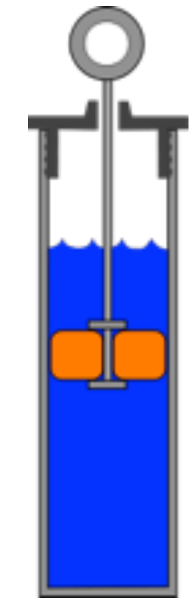

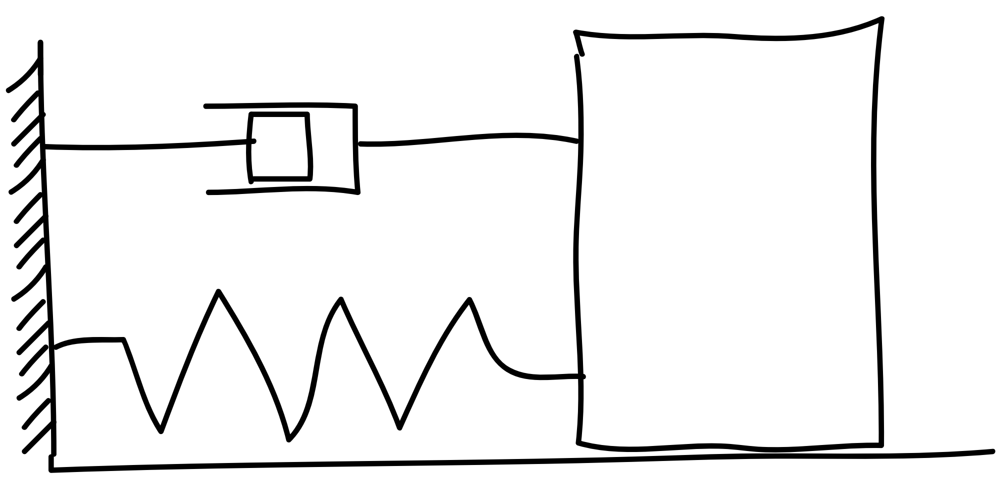

Kelvin-Voigt model

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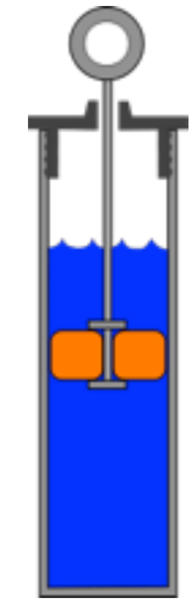

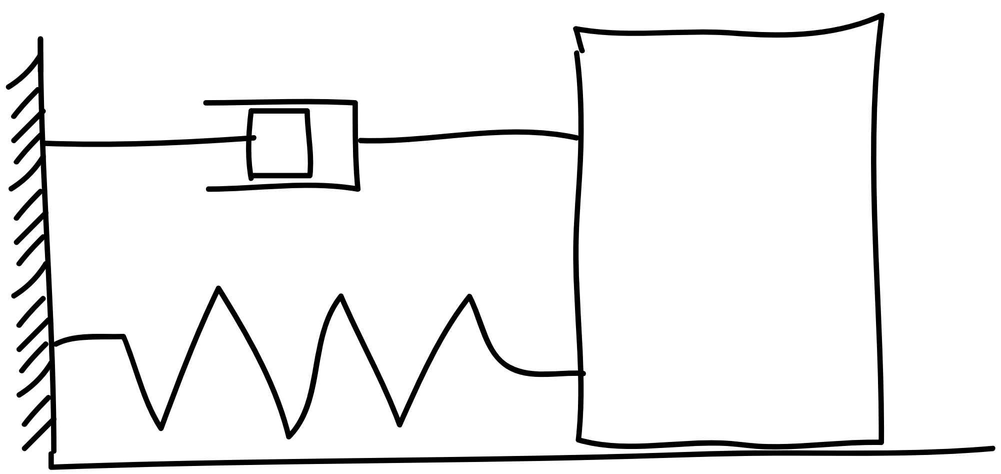

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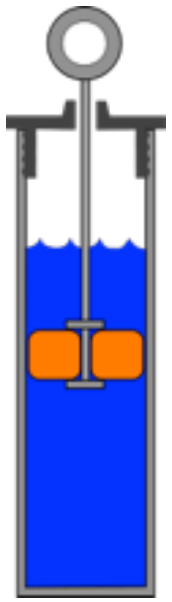

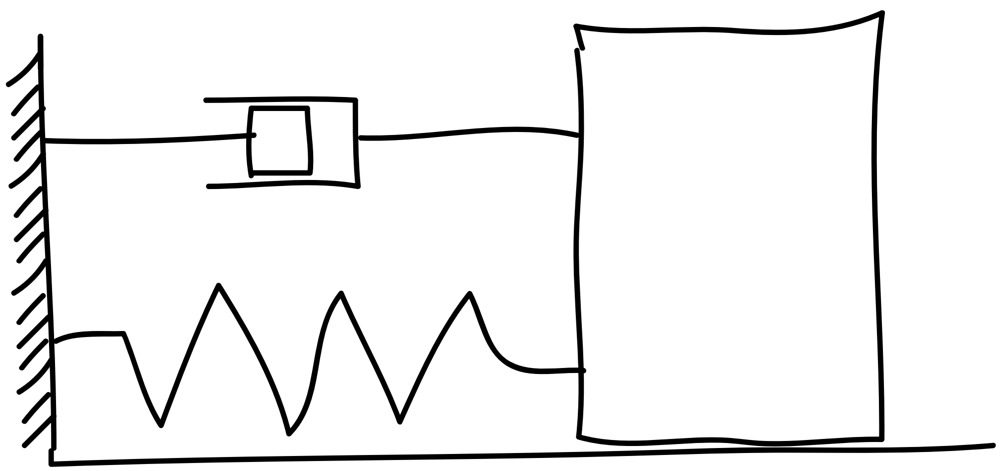

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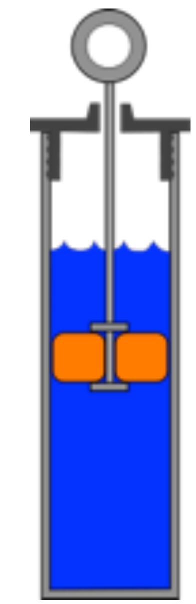

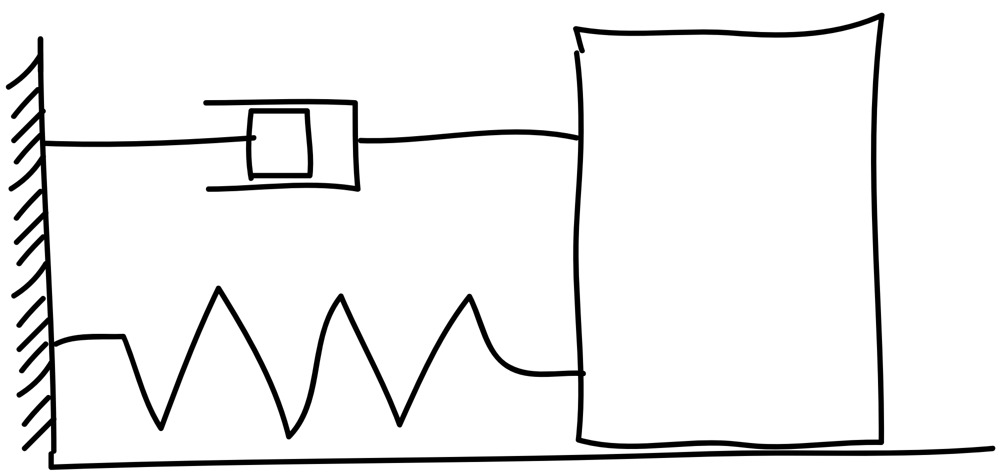

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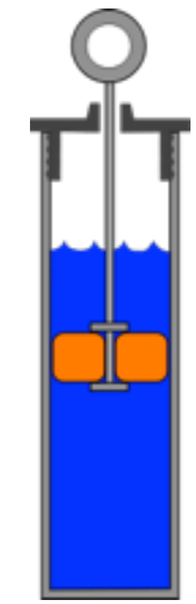

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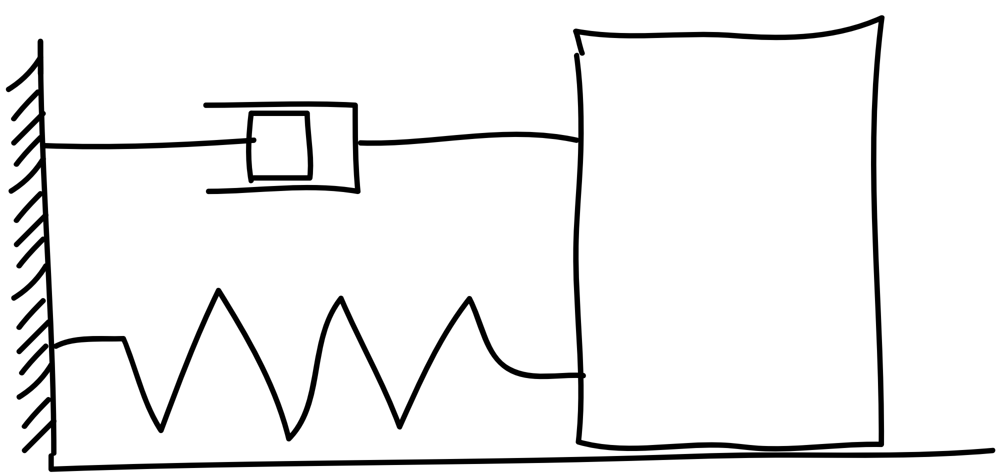

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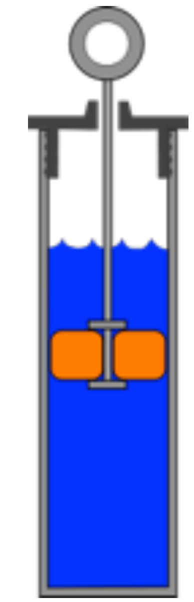

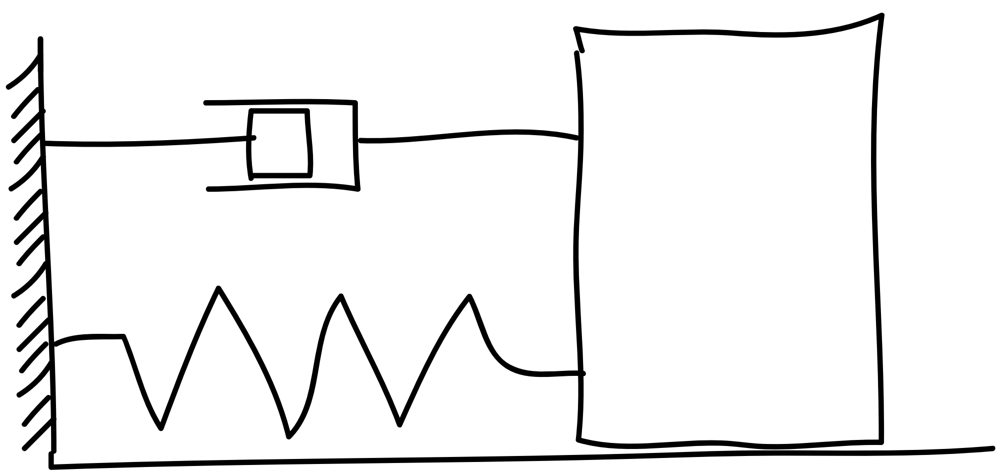

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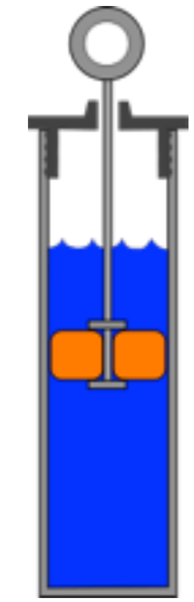

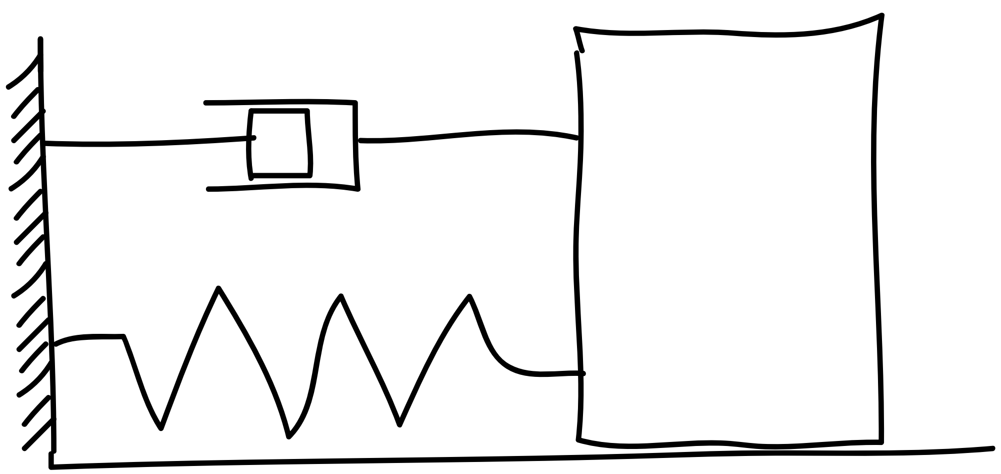

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Applications - forced vibrations

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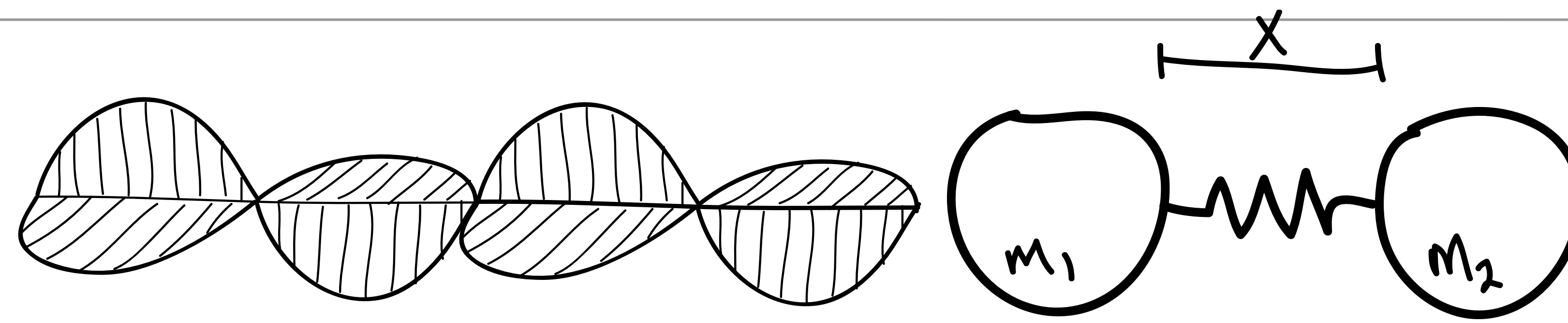

- light hitting a molecular bond

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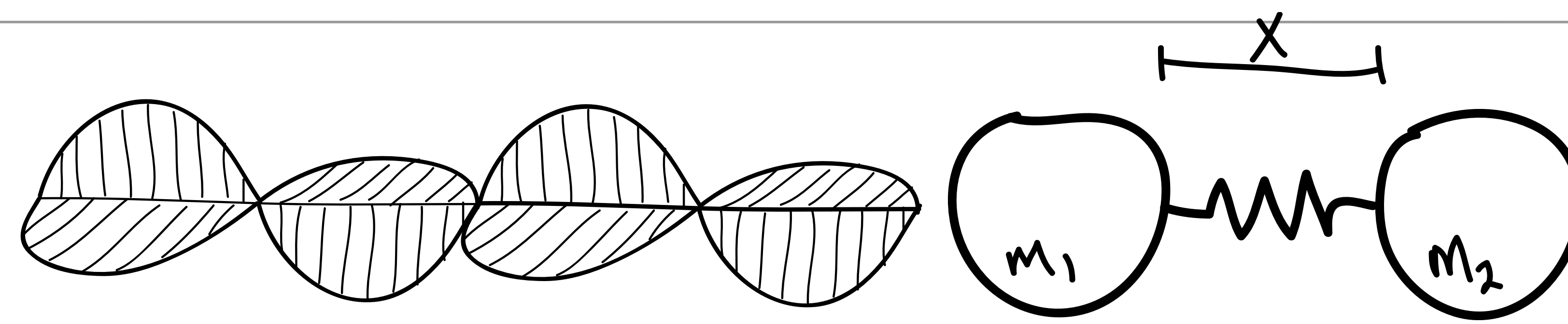
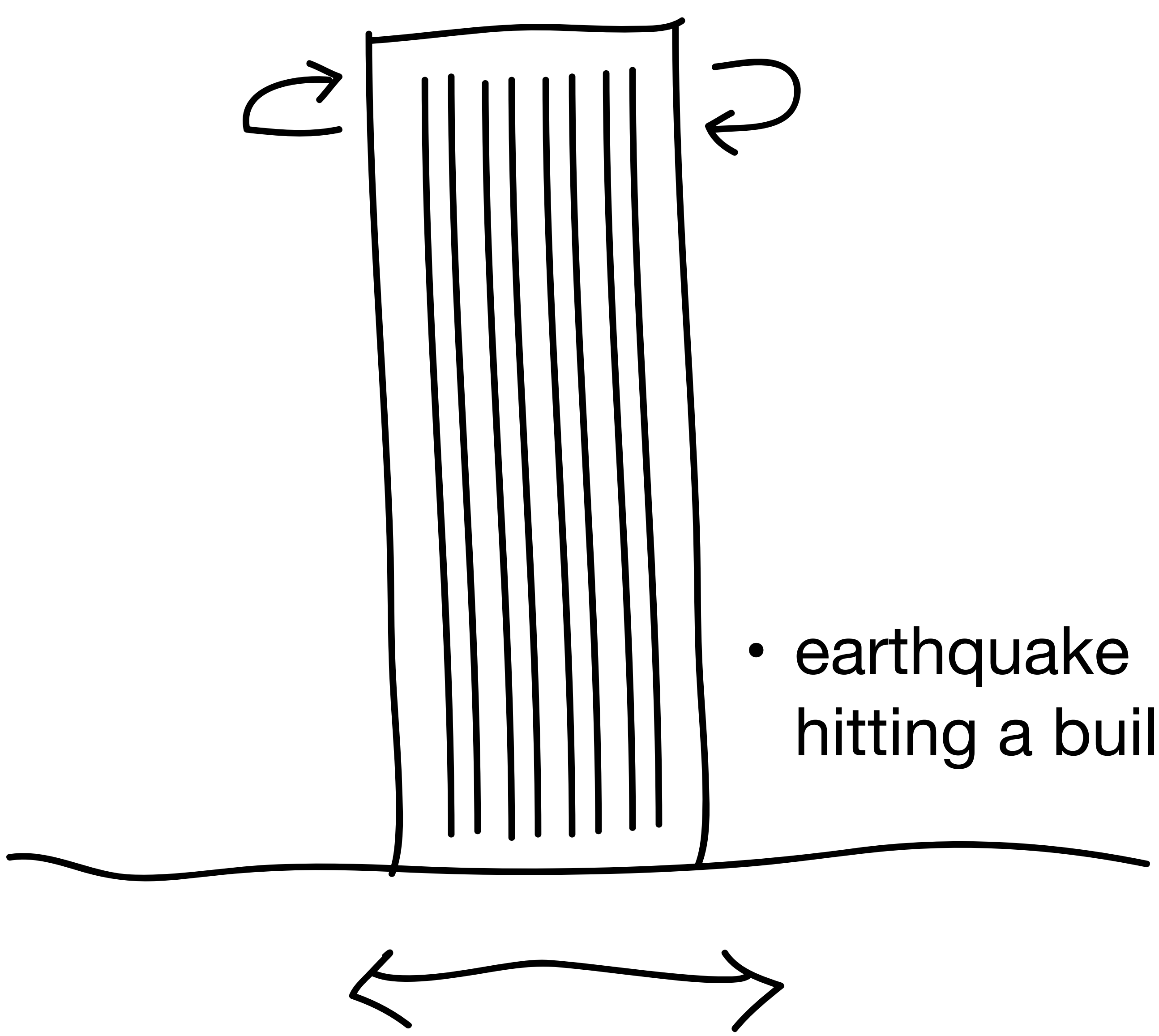


- light hitting a molecular bond



- earthquake
  hitting a building

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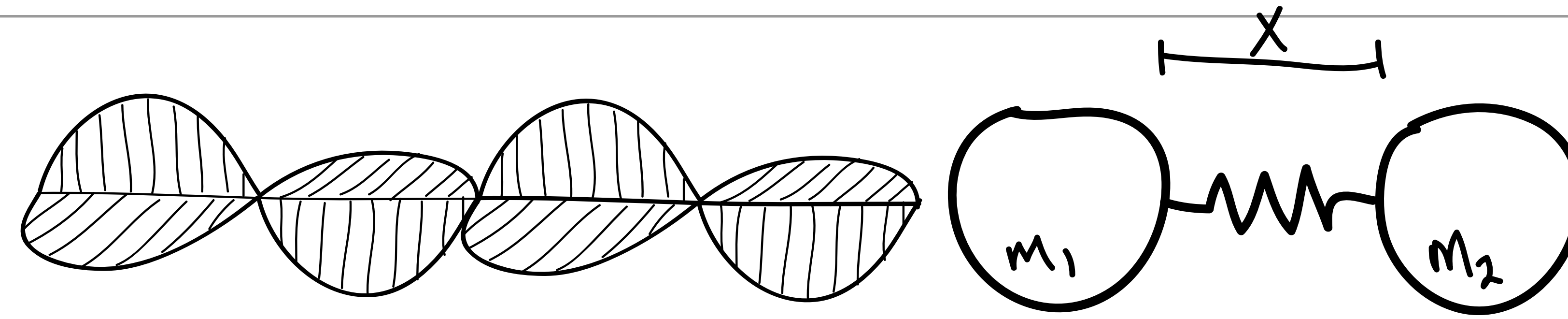
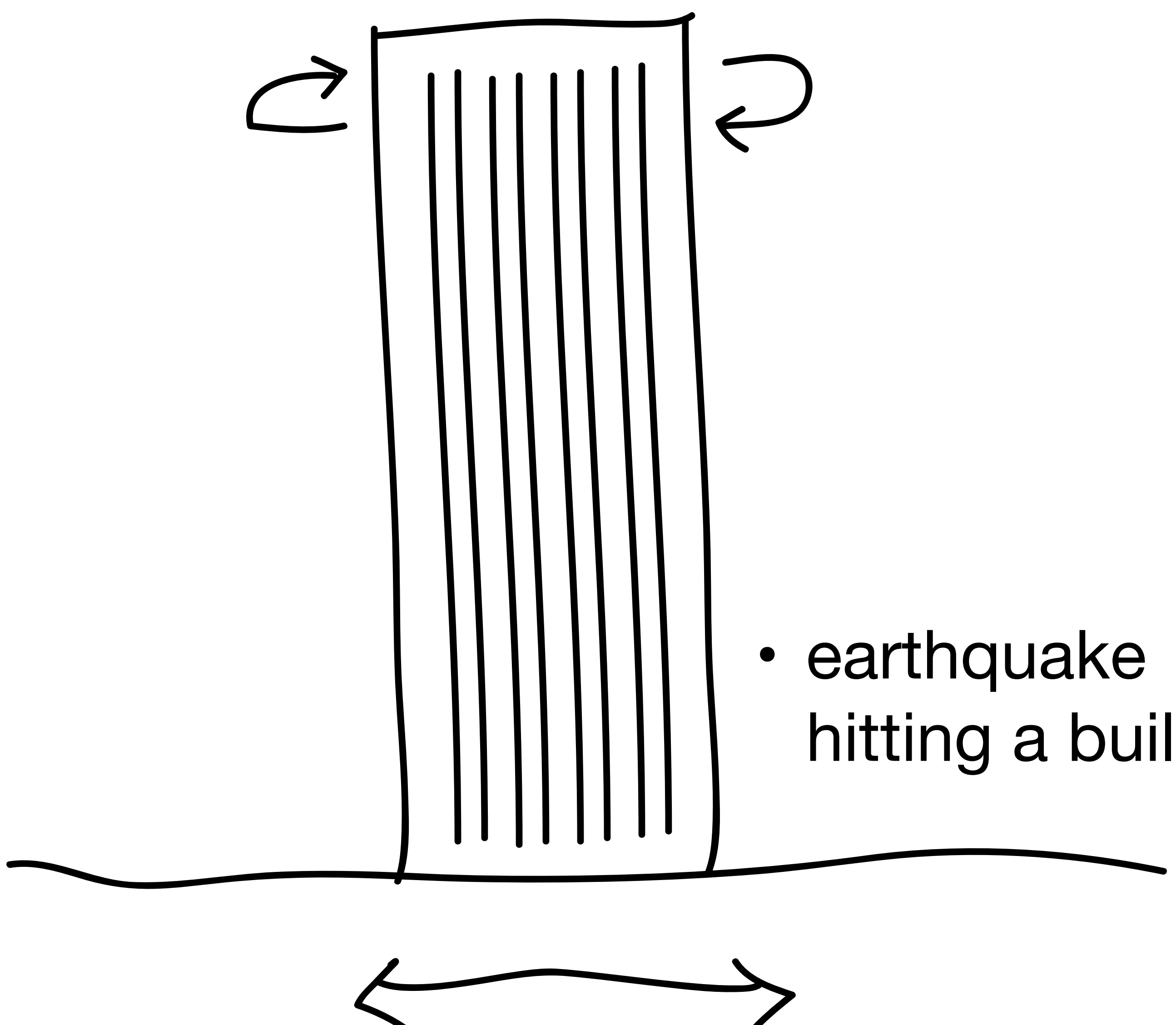
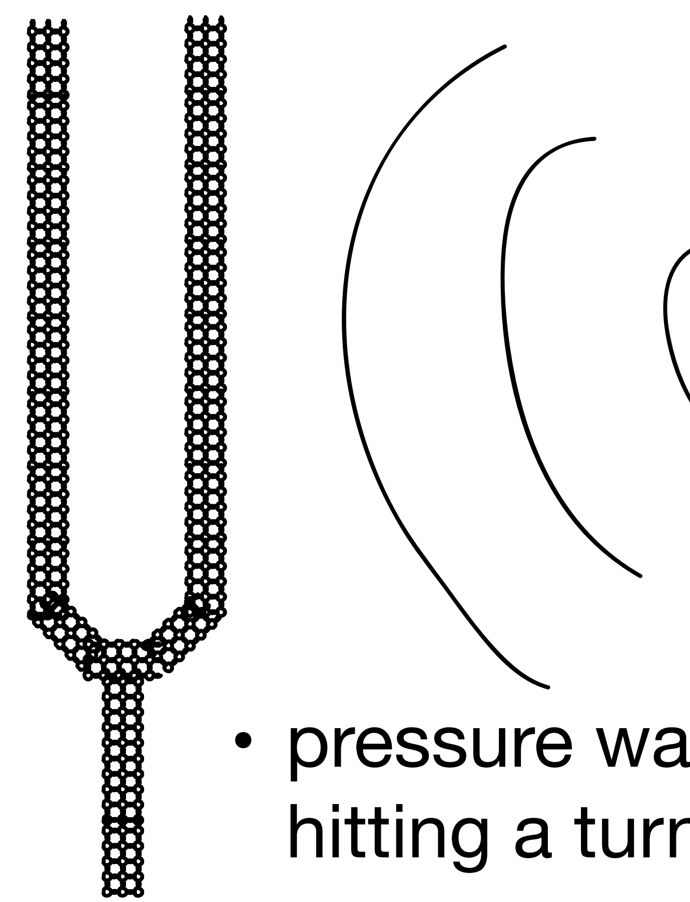


- light hitting a molecular bond



- earthquake hitting a building



- pressure waves (sound) hitting a turning fork.