

Full Name: _____
Student Number: _____

Math 103-202 Midterm Test 2 Mar. 17, 2006 50 min.

1. Ensure that **your *full* name and student number** appear on this page.
2. No calculators, books, or notes, or electronic devices of any kind are permitted.
3. Unless otherwise indicated, show all your work. Answers not supported by calculations or reasoning may not receive credit. Correct answers that do not demonstrate an appropriate understanding of the course material might not receive credit. Messy work will not be graded. **Read each question carefully** to be sure you are answering the question being asked.
4. Exposing your test paper, copying from another student's paper, or sharing information about this test constitutes academic dishonesty. Such behaviour may jeopardize your grade on this test, in this course, and your standing at this university.
5. Five minutes before the end of the test period you will be given a verbal notice. After that time, you must remain seated until all test papers have been collected.
6. When the test period is over, you will be instructed to put away writing implements. Put away all pens and pencils at this point. Continuing to write past this instruction will be considered as cheating.
7. Please remain seated and pass your test paper down the row to the nearest indicated aisle. Once all the test papers have been collected, you are free to leave.

Problem #	Grade	Value
1	15	
2	8	
3	7	
Total	30	

I have read and understood the instructions and agree to abide by them.

Signed: _____

Formulae

$$\sin^2 \theta + \cos^2 \theta = 1, \quad \tan^2 \theta + 1 = \sec^2 \theta,$$

$$\sin(A + B) = \sin A \cos B + \sin B \cos A, \quad \cos(A + B) = \cos A \cos B - \sin A \sin B,$$

$$\cos^2 \theta = \frac{1 + \cos 2\theta}{2}, \quad \sin^2 \theta = \frac{1 - \cos 2\theta}{2}$$

$$\int \sec \theta d\theta = \ln |\sec \theta + \tan \theta| + C, \quad \int \sec^3 \theta d\theta = \frac{1}{2} (\sec \theta \tan \theta + \ln |\sec \theta + \tan \theta|) + C$$

$$\frac{d}{dx}(\tan x) = \sec^2 x, \quad \frac{d}{dx}(\cot x) = -\csc^2 x, \quad \frac{d}{dx}(\sec x) = \tan x \sec x, \quad \frac{d}{dx}(\csc x) = -\cot x \csc x,$$

$$\int \frac{1}{1+u^2} du = \arctan u + C, \quad \int \frac{1}{\sqrt{1-u^2}} du = \arcsin u + C$$

$$\text{Expected value of } x = \mathbf{E}x = \sum_{k=1}^n x_k p(x = x_k)$$

$$\text{Variance of } x = \sum_{k=1}^n (x_k - \mathbf{E}x)^2 p(x = x_k)$$

$$\text{Standard deviation of } x = \sqrt{\text{Variance of } x}$$

$$p(E_1 \text{ and } E_2) = p(E_1)p(E_2|E_1), \quad p(E_1 \text{ or } E_2) = p(E_1) + p(E_2) - p(E_1 \text{ and } E_2)$$

$$C(n, k) = \frac{n!}{(n-k)!k!}, \quad P(n, k) = \frac{n!}{(n-k)!}$$

$$P(k \text{ heads out of } n \text{ tosses}) = C(n, k)p^k(1-p)^{n-k}$$

1. Calculate the following (write your answer in the box – these questions will be graded simply right or wrong).

(a) $\int e^{-x} dx$.

- (b) The probability of getting 4 “tails” in 6 tosses of a biased coin with $P(\text{“heads”}) = 2/3$.

- (c) The total mass of a wire lying between $x = 0$ and $x = \pi/2$ if its density (in kg/m) is

$p(x) = \sqrt{\cos(x)} \sin(x)$.

- (d) The probability of rolling a total of 6 with 3 dice.

- (e) The derivative of $f(x) = \int_2^x te^t dt$.

- (f) The number of distinct words (meaningless ones allowed) that can be made by rearranging the letters of the word “textbook”.

- (g) The area under the graph $y = \frac{1}{4-x^2}$ for $0 \leq x \leq 1$.

- (h) The amount you expect to win (or lose) (i.e. the expected value of the amount you win) in the following game: 3 fair coins are tossed, and you win \$8 if exactly 2 “heads” are showing. Otherwise you lose \$4.

- (i) $\int \sec^4(x) dx$.

- (j) The probability of getting exactly 2 aces when you draw 3 cards from a standard deck of 52 cards.

2. A wire lying between $x = 0$ and $x = 1$ on the x -axis has density $p(x) = e^x$.

- (a) Which part of the wire is most dense?
(b) Where should the wire be cut to divide it into two pieces of equal mass?

- (c) Find the centre of mass of the wire.
3. You randomly paint the 5 fingernails of your right hand 5 different colours (red, green, pink, blue, and purple). Find the probability of
- (a) your thumb being painted green
 - (b) your thumb green and your pinky purple
 - (c) your thumb green or your pinky purple
 - (d) your green and purple fingers being next to each other