

Full Name: \_\_\_\_\_  
Student Number: \_\_\_\_\_

**Math 103-201 Midterm Test 2    March 17, 2006    50 min.**

1. Ensure that **your *full name and student number*** appear on this page.
2. No calculators, books, or notes, or electronic devices of any kind are permitted.
3. Unless otherwise indicated, show all your work. Answers not supported by calculations or reasoning may not receive credit. Correct answers that do not demonstrate an appropriate understanding of the course material might not receive credit. Messy work will not be graded. **Read each question carefully** to be sure you are answering the question being asked.
4. Exposing your test paper, copying from another student's paper, or sharing information about this test constitutes academic dishonesty. Such behaviour may jeopardize your grade on this test, in this course, and your standing at this university.
5. Five minutes before the end of the test period you will be given a verbal notice. After that time, you must remain seated until all test papers have been collected.
6. When the test period is over, you will be instructed to put away writing implements. Put away all pens and pencils at this point. Continuing to write past this instruction will be considered as cheating.
7. Please remain seated and pass your test paper down the row to the nearest indicated aisle. Once all the test papers have been collected, you are free to leave.

Problem #	Grade	Value
1		16
2		6
3		8
Total		30

**I have read and understood the instructions and agree to abide by them.**

Signed: \_\_\_\_\_

## Formulae

$$\sin^2 \theta + \cos^2 \theta = 1, \quad \tan^2 \theta + 1 = \sec^2 \theta, \quad \cot^2 \theta + 1 = \csc^2 \theta,$$

$$\sin(A + B) = \sin A \cos B + \sin B \cos A, \quad \cos(A + B) = \cos A \cos B - \sin A \sin B,$$

$$\cos^2 \theta = \frac{1 + \cos 2\theta}{2}, \quad \sin^2 \theta = \frac{1 - \cos 2\theta}{2}$$

$$\int \sec \theta d\theta = \ln |\sec \theta + \tan \theta| + C, \quad \int \sec^3 \theta d\theta = \frac{1}{2} (\sec \theta \tan \theta + \ln |\sec \theta + \tan \theta|) + C$$

$$\int \frac{1}{1+u^2} du = \arctan u + C, \quad \int \frac{1}{\sqrt{1-u^2}} du = \arcsin u + C$$

$$\frac{d}{dx}(\tan x) = \sec^2 x, \quad \frac{d}{dx}(\cot x) = -\csc^2 x, \quad \frac{d}{dx}(\sec x) = \tan x \sec x, \quad \frac{d}{dx}(\csc x) = -\cot x \csc x,$$

$$\text{Expected value of } x = \mathbf{E}x = \sum_{k=1}^n x_k p(x = x_k)$$

$$\text{Variance of } x = \sum_{k=1}^n (x_k - \mathbf{E}x)^2 p(x = x_k)$$

$$\text{Standard deviation of } x = \sqrt{\text{Variance of } x}$$

$$p(E_1 \text{ and } E_2) = p(E_1)p(E_2|E_1), \quad p(E_1 \text{ or } E_2) = p(E_1) + p(E_2) - p(E_1 \text{ and } E_2)$$

$$C(n, k) = \frac{n!}{(n-k)!k!}, \quad P(n, k) = \frac{n!}{(n-k)!}$$

$$P(k \text{ heads out of } n \text{ tosses}) = C(n, k)p^k(1-p)^{n-k}$$

1. Place your answer to each question in the box. Your answer will be marked right or wrong (work will not be considered for this section).

(a) Calculate  $\int x \cos(x^2) dx$ .

- (b) What is probability that the sum of two rolls of a die is an odd number?

(c) Calculate  $\int_{-1}^1 \sqrt{1+x^2} dx$ .

- (d) If the probability of rain on any day in February was 0.6, what is the probability of it having rained on a total of exactly 26 days during that month? Reminder: February had 28 days this year. You do not have to reduce the expression you get.

(e) Calculate  $\int \frac{1}{x^2+6x+10} dx$ .

(f) How many distinct words (potentially meaningless) can be made by rearranging the letters of the word “anagram” ? (e.g. granama, naamarg ...)

(g) Calculate  $\int_{\pi/4}^{\pi/2} \csc^4(x) dx$ .

(h) Consider a game in which you win if two dice, when rolled, show a total of 2. Each time you win you get \$35 dollars and each time you lose, you lose \$1. If you play 108 times, overall, how much do expect to win? (A loss should be denoted by a negative number.)

2. A rod with ends at  $x = 0$  cm and  $x = 1$  cm has a (linear) density given by  $\frac{1}{1+x^2}$  grams per cm.
- (a) Find the total mass of the rod.
  - (b) At what point should the rod be cut so that the two resulting pieces have equal weight?
  - (c) At what point is the center of mass of the rod?

3. A child is playing a game in which 5 coloured beads must be hung vertically on a string, one above the other. Each bead is a different colour. The child does not pay attention to the colours while placing them on the string. Find the probability of finding
- (a) the blue bead on the bottom and the green bead at the top,
  - (b) the orange bead second from the bottom or the blue bead at the top,
  - (c) the green and purple beads not next to each other,
  - (d) the red bead above both the green and blue beads.