

Full Name: _____
Student Number: _____

Math 103 Midterm Test 1 Feb. 10, 2006 50 min.

1. Ensure that **your full name and student number** appear on this page.
2. No calculators, books, or notes, or electronic devices of any kind are permitted.
3. Unless otherwise indicated, show all your work. Answers not supported by calculations or reasoning may not receive credit. Correct answers that do not demonstrate an appropriate understanding of the course material might not receive credit. Messy work will not be graded. **Read each question carefully** to be sure you are answering the question being asked.
4. Exposing your test paper, copying from another student's paper, or sharing information about this test constitutes academic dishonesty. Such behaviour may jeopardize your grade on this test, in this course, and your standing at this university.
5. Five minutes before the end of the test period you will be given a verbal notice. After that time, you must remain seated until all test papers have been collected.
6. When the test period is over, you will be instructed to put away writing implements. Put away all pens and pencils at this point. Continuing to write past this instruction will be considered as cheating.
7. Please remain seated and pass your test paper down the row to the nearest indicated aisle. Once all the test papers have been collected, you are free to leave.

Problem #	Grade	Value
1		10
2		6
3		6
4		8
Total		30

I have read and understood the instructions and agree to abide by them.

Signed: _____

Summation Formulae

$$\sum_{k=1}^N C = NC \qquad \sum_{k=1}^N k = N(N+1)/2 \qquad \sum_{k=1}^N k^2 = N(N+1)(2N+1)/6$$
$$\sum_{k=1}^N k^3 = (N(N+1)/2)^2 \qquad \sum_{k=0}^N r^k = (1-r^{N+1})/(1-r)$$

1. Calculate each of the following.

(a) $\sum_{k=3}^{15} 2^k.$

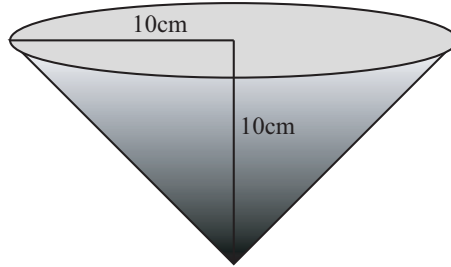
(b) $\int_0^2 e^{-2x} dx.$

(c) Find the area under the curve $f(x) = x^2$ between $x = -1$ and $x = 2$.

(d) The area between the graphs $y = 2 - x^2$ and $y = |x|$.

(e) An approximation for the sum $\sum_{k=1}^{40} \sin\left(\frac{\pi}{80}k\right)$. Is your approximation higher or lower than the actual value?

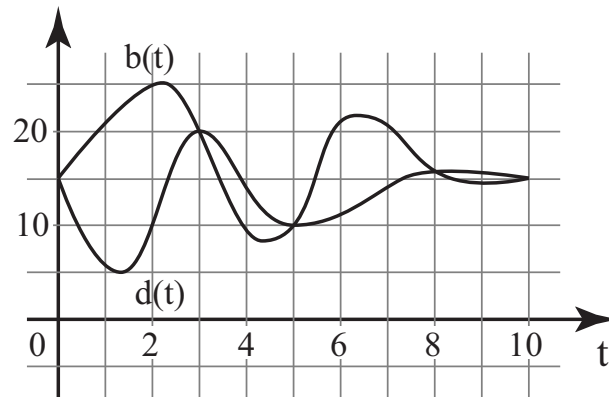
2. A cone-shaped coffee filter with a height of 10 cm and an opening radius of 10 cm is used to make coffee. While in use, the filter is full of water and the density of coffee grounds is given by $\rho(y) = -\frac{1}{10}y$ where $y = 0$ corresponds to the top of the filter. y is measured in cm and ρ is measured in grams/cm³.



- (a) Use the method of disks to find the volume of water in the filter.
(b) Calculate the total mass of coffee grounds in the filter.

3. A sink can hold a maximum of 20 liters before it overflows. Suppose the sink is initially empty. The inflow rate from the tap is given by $T(t) = A(1 + e^{-3t})$ and the outflow through the drain is $D(t) = A(1 - e^{-4t})$. $T(t)$ and $D(t)$ are both measured in liters per minute and t is measured in minutes.
- (a) How many liters of water are in the sink at time t ?
 - (b) What is the maximum value of A for which the sink does not eventually overflow?

4. Below are graphs of the birth rate and death rate of a particular species of wild hare in the arctic over a period of ten years starting in 1995 ($x = 0$). By examining the graphs, answer the following questions.



- (a) Determine the years in which the population size was increasing and in which years the population size was decreasing.
- (b) At the start of which year was the population of hares largest and when was it smallest? Briefly explain your answer.