

The University of British Columbia

Final Examination - December 16, 20xx

Mathematics 104/184

All Sections

Closed book examination

Time: 2.5 hours

Last Name _____ First _____ Signature _____

Student Number _____

Special Instructions:

No books, notes, or calculators are allowed.

Student Conduct during Examinations

- Each examination candidate must be prepared to produce, upon the request of the invigilator or examiner, his or her UBCcard for identification.
- Candidates are not permitted to ask questions of the examiners or invigilators, except in cases of supposed errors or ambiguities in examination questions, illegible or missing material, or the like.
- No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination. Should the examination run forty-five (45) minutes or less, no candidate shall be permitted to enter the examination room once the examination has begun.
- Candidates must conduct themselves honestly and in accordance with established rules for a given examination, which will be articulated by the examiner or invigilator prior to the examination commencing. Should dishonest behaviour be observed by the examiner(s) or invigilator(s), pleas of accident or forgetfulness shall not be received.
- Candidates suspected of any of the following, or any other similar practices, may be immediately dismissed from the examination by the examiner/invigilator, and may be subject to disciplinary action:
 - (a) speaking or communicating with other candidates, unless otherwise authorized;
 - (b) purposely exposing written papers to the view of other candidates or imaging devices;
 - (c) purposely viewing the written papers of other candidates;
 - (d) using or having visible at the place of writing any books, papers or other memory aid devices other than those authorized by the examiner(s); and,
 - (e) using or operating electronic devices including but not limited to telephones, calculators, computers, or similar devices other than those authorized by the examiner(s)–(electronic devices other than those authorized by the examiner(s) must be completely powered down if present at the place of writing).
- Candidates must not destroy or damage any examination material, must hand in all examination papers, and must not take any examination material from the examination room without permission of the examiner or invigilator.
- Notwithstanding the above, for any mode of examination that does not fall into the traditional, paper-based method, examination candidates shall adhere to any special rules for conduct as established and articulated by the examiner.
- Candidates must follow any additional examination rules or directions communicated by the examiner(s) or invigilator(s).

1		42
2		10
3		12
4		12
5		12
6		12
Total		100

[42] **1. Short Problems.** Each question is worth 3 points. Put your answer in the box provided and show your work. No credit will be given for the answer without the correct accompanying work.

(a) Find the number c that makes

$$f(x) = \begin{cases} \frac{x^2 - 3x - 10}{x + 2} & \text{if } x \neq -2 \\ c & \text{if } x = -2 \end{cases}$$

continuous for every x .

Answer:

(b) Evaluate $\lim_{t \rightarrow 0} \frac{\sqrt{t+9} - 3}{\sqrt{t}}$.

Answer:

(c) Evaluate $\lim_{h \rightarrow 0} \frac{(h+3)^2 - 9}{(h-5)^2 - 25}$.

Answer:

(d) If you put money in an account that pays 6% interest, compounded continuously, how long will it take for your money to triple?

Answer:

- (e) Find the x and y coordinates of a point on the graph of $y = \frac{1}{4}(2x + 1)^2$ where the tangent line is parallel to the line $y - 3x = 1$.

Answer:

- (f) For what x does the graph of $y = e^{3x} + e^{-2x}$ have slope zero?

Answer:

- (g) Let $f(x) = (\sin^{-1} x)^{-2}$, where $\sin^{-1} x$ is the inverse sine function. Find $f' \left(\frac{1}{\sqrt{2}} \right)$. Leave your answer in “calculator-ready” form.

Answer:

- (h) If $f(x)$ is a function satisfying $f(0) = 1$ and $f'(0) = 4$, find the equation of the tangent line to the graph of $g(x) = \sqrt{1 + 3f(x)}$ at $x = 0$.

Answer:

- (i) Let $y = y(x)$ be the function defined implicitly by the equation $y + \ln(y + 3) = x^2$. Find $\frac{dy}{dx}$ in terms of x and y .

Answer:

- (j) Determine where the function

$$f(x) = \frac{1 + \ln(x + 1)}{x + 1}, \quad x > -1$$

is increasing.

Answer:

- (k) Determine where the function $f(x) = \frac{x}{x + 2}$, $x \neq -2$, is concave down.

Answer:

(l) Find the global minimum of the function $f(x) = \frac{x}{x^2 + 4}$ over the interval $[-1, 5]$.

Answer:

(m) Determine the sum $-4\left(\frac{1}{5}\right) + 4\left(\frac{1}{5}\right)^2 - 4\left(\frac{1}{5}\right)^3 + 4\left(\frac{1}{5}\right)^4 - \dots$.

Answer:

(n) Let $c_0 + c_1x + c_2x^2 + \dots$ be the Taylor Series of the function $f(x) = (x + 1)e^{-x}$ at $a = 0$. Determine the value of c_2 .

Answer:

Long Problems. In questions 2 - 6, show your work. No credit will be given for the answer without the correct accompanying work.

[10] **2.** Use the definition of the derivative as a limit to find the derivative of $f(x) = \frac{x}{1-3x}$, $x \neq \frac{1}{3}$. No marks will be given for the use of differentiation rules.

[12] **3.** Suppose that when a busy restaurant charges \$7 for its tomato appetizer, an average of 60 people order the dish each night. When it drops the price of the appetizer to \$5, the number ordering it rises to 66. Assume that the demand q is a linear function of the price p . If each appetizer costs the restaurant \$3 to make, use calculus to find the price it should charge to maximize its profit from the appetizer. You do not need to justify that your answer provides the maximum profit.

[12] 4. Let $y = f(x) = \frac{2x}{x^2 + 1}$.

- (a) Find the interval or intervals on which $f(x)$ is increasing. [3pts]
- (b) Find the interval or intervals on which $f(x)$ is concave up. [3pts]
- (c) Sketch the graph of $y = f(x)$, and indicate the x -coordinates of any inflection points, and the values of x where any global maxima or global minima occur. (Hint: $\sqrt{3} \approx 1.7$) [6pts]

[12] 5. A carpenter has been asked to build an open box with a square base, where an open box means a box without a top. The sides of the box will cost \$2 per square meter, and the base will cost \$5 per square meter. What are the dimensions of the box of maximal volume that can be constructed for \$60 ? You do not need to justify that your answer provides the maximal volume.

[12] 6. The price p (in dollars) and demand q for a product are related by

$$p^2 + 2q^2 = 1100.$$

If the price is increasing at a rate of \$2 per month when the price is \$30, find the rate of change of the revenue in dollars per month.