

## Our approach so far...

Solving related rates problems requires us to do the following actions.

- Make sense of the situation at hand
- Remember a model that you know applies here OR make sense of the model given
- identify all the variables at play and their role (functions, constants, actual variable (usually time))
- find the relationship(s) linking the variables of interest
- write these relationships as an equation making explicit which variables are functions or constants
- take the derivative on both sides of the equation(s)
- solve for the desired rate of change
- compute the desired rate of change by substituting all the known values and computing the missing values if necessary
- answer the problem by making the correct interpretation of your computations

Your notes:

Example 1:

Find  $\frac{dh}{dt}$  when  $t=250$

$$V(t) = 250h^2$$

$$\text{given } \frac{dV}{dt} = 1$$

$$\text{used } h = 1$$

Find  $h'(250)$

$$V(t) = 250(h(t))^2$$

$$\text{given } V'(t) = 1$$

$$\text{used } h(250) = 1 \text{ (true)}$$


Example 2: Find  $\frac{dD}{dt}$  when  $t=30$

$$\text{used } x(30), y(30), \cancel{x'(30)}, \cancel{y'(30)}$$

$$\text{used } x, y, \frac{dx}{dt}, \frac{dy}{dt}$$

Find  $D'(30)$

$$\text{used } x(30), y(30), x'(30), y'(30)$$

Problem A	Problem B	To do:	To compare:
<p>(From homework and today) Two boats leave a port at the same time, one traveling west and the other traveling south. At the moment where the first one is 15 kilometres away from the port with a speed of 20 kilometres per hour, the other boat is 20 kilometres away and has a speed of 30 kilometres per hour. At what rate is the distance between the two boats changing?</p>	<p>Two boats leave a port at the same time, one traveling east and the other traveling south. When the boat heading east is 10 kilometres away from the port, the boat heading south reaches 20 kilometres away from the port and stops there for the day. When the boat heading east is 15 kilometres away from the port with a speed of 20 kilometres per hour, at what rate is the distance between the two boats changing?</p>	<p>In Problem B, find the relation between the rates (the derivative equation) and solve for the target rate (no need to plug in numbers).</p> <p><i>Prob A</i></p>  $x^2 + y^2 = z^2$ $2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 2z \frac{dz}{dt}$ $z'(t) = \frac{x(t) \frac{dx}{dt} + y(t) \frac{dy}{dt}}{z(t)}$	<p>What is the key difference between Problems A and B?</p> <p><math>y'(t) = \frac{dy}{dt}</math> (same thing).</p> <p><math>y'(t) = 0</math> for south-travelling boat in Prob B.</p> <p>i.e. <math>y</math> constant in time of interest.</p>
<p><b>Problem C</b></p> <p>(From today) Let us denote by <math>q</math> the number of shirts demanded that we sell for a price of <math>p</math> dollars per unit and assume we have the following model</p> $pq - 30p - 2000 = 0$ <p>If the shirt is selling at \$25 at the moment and is increasing at a rate of \$0.20 per month, what will the model predict about the rate of change of demand?</p>	<p><b>Problem D</b></p> <p>(From homework, almost) A snowball is melting under the sun. At some precise instant, we take two measurements and discover that its volume is 40 cubic centimetres and its volume is changing at the rate of 10 cubic centimetres per minute. At what rate is the radius of the snowball changing at that moment? You may use the fact that the volume <math>V</math> of a sphere with radius <math>r</math> is given by</p> $V = \frac{4}{3} \pi r^3$	<p>To do:</p> <p>In Problem D, find the relation between the rates (the derivative equation) and solve for the target rate (no need to plug in numbers).</p> $\frac{dV}{dt} = \frac{4\pi}{3} \cdot 3r^2 \frac{dr}{dt}$ $\frac{dV}{dt} = \frac{3}{4\pi r^2} \cdot \frac{dV}{dt}$ <p>use <math>r = r(t) = ?</math> <math>\frac{dV}{dt} = -10</math></p>	<p>To compare:</p> <p>In terms of the work to do, how are these two problems similar? How do they differ from A and B?</p> <p>Problem C has no geometric model.</p> <p>Problem D has a given model.</p> <p>Both C, D relate two quantities while A, B relate three.</p>



<p><b>Problem E</b></p> <p>(From homework) Consider a tank of water that is draining by a hole at its bottom. The tank has the shape of an inverted cone that is 5 metres high and that has a base radius of 3 metres. At the precise moment when the water level is 2 metres, we measure that the tank is draining at a rate of 0.7 cubic metres per second. At that instant, what is the speed at which the water level is decreasing?</p>	<p><b>To do:</b></p> <p>Solve Problem E:</p> <p>see pencast. + class notes.</p>	<p><b>To compare:</b></p> <p>(From homework) What makes Problem E different from the group of Problems A-D?</p> <p>Need to eliminate one of the three quantities. Pair of relations: V to r and h and r to h alone.</p>
<p><b>Problem F</b></p> <p>Consider a tank of water that is draining by a hole at its bottom. The tank has the shape of a cylinder that is 5 metres high and that has a radius of 3 metres. At the precise moment at which the water level is 2 metres, we measure that the tank is draining at a rate of 0.7 cubic metres per second. At that instant, what is the speed at which the water level is decreasing?</p>	<p><b>To do:</b></p> <p>Solve Problem F:</p>	<p><b>To compare:</b></p> <p>What is the key difference between Problems E and F?</p> <p><math>\frac{dr}{dt} = 0</math> and <math>\frac{dr}{dh} = 0</math></p> <p>Radius is constant for a cylinder!</p>

<p><b>Problem G</b></p> <p><i>(Related to homework)</i> A ladder that is 5 metres long is leaning against a wall. If I am pulling the bottom of the ladder away from the wall at a rate of 2 metres per second, how quickly is the top of the ladder moving if the top of the ladder is 4 metres from the floor?</p>	<p><b>To do:</b></p> <p>Solve Problem G:</p>	<p><b>To compare:</b></p> <p>What is different about the use of triangles in this and the other problems?  <i>Hypotenuse is of constant length.</i></p>
<p><b>Problem H</b></p> <p>A ladder that is 5 metres long is leaning against a wall. If I am pulling the bottom of the ladder away from the wall at a constant rate of 2 metres per second, how fast are the top and bottom of the ladder moving when they are moving with the same speed?</p>	<p><b>To do:</b></p> <p>Solve Problem H:</p>	<p><b>To compare:</b></p> <p>What makes this different from the other related rates problems?</p>