Logarithmic Differentiation

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1)

$$f(x) = \left(1 + \frac{1}{x}\right)^x$$

$$\ln(f(x)) = \ln\left(\left(1 + \frac{1}{x}\right)^x\right)$$

$$\ln(f(x)) = x \ln\left(1 + \frac{1}{x}\right)$$

$$\frac{f'(x)}{f(x)} = \frac{d}{dx}\left(x \ln\left(1 + \frac{1}{x}\right)\right)$$

$$\frac{f'(x)}{f(x)} = \ln\left(1 + \frac{1}{x}\right) + x \cdot \frac{1}{1 + \frac{1}{x}}\left(-x^{-2}\right)$$

$$\frac{f'(x)}{f(x)} = \ln\left(1 + \frac{1}{x}\right) - \frac{x}{x^2 + x}$$

$$f'(x) = f(x)\left(\ln\left(1 + \frac{1}{x}\right) - \frac{1}{x+1}\right)$$

$$f'(x) = \left(1 + \frac{1}{x}\right)^x\left(\ln\left(1 + \frac{1}{x}\right) - \frac{1}{x+1}\right)$$

2)

$$g(x) = (\ln(x))^{x^2}$$

$$\ln(g(x)) = x^2 \ln(\ln(x))$$

$$\frac{g'(x)}{g(x)} = 2x \ln(\ln(x)) + x^2 \frac{1}{\ln(x)} \cdot \frac{1}{x}$$

$$g'(x) = g(x)\left(2x \ln(\ln(x)) + \frac{x}{\ln(x)}\right)$$

$$g'(x) = (\ln(x))^{x^2}\left(2x \ln(\ln(x)) + \frac{x}{\ln(x)}\right)$$

(3) $h(x) = \dfrac{\sin^{10}(x)}{(5x+3)^6}$ could use quotient rule.

$$\ln(h(x)) = \ln(\sin^{10}(x)) - \ln((5x+3)^6)$$

$$\ln(h(x)) = 10\ln(\sin(x)) - 6\ln(5x+3)$$

$$\frac{h'(x)}{h(x)} = 10 \cdot \frac{1}{\sin(x)} \cdot \cos(x) - 6 \cdot \frac{1}{5x+3} \cdot 5$$

$$h'(x) = h(x)\left(10\frac{\cos(x)}{\sin(x)} - \frac{30}{5x+3}\right)$$

$$h'(x) = \frac{\sin^{10}(x)}{(5x+3)^6}\left(10\frac{\cos(x)}{\sin(x)} - \frac{30}{5x+3}\right)$$