Math 104 - Section 103

Last Name Solutions **First Name**

Student Number

1. Estimate $e^{0.5}$ by using the linear approximation to the function $f(x) = e^x$ at $a = 0$.

$f'(x) = e^x$ $f(0) = 1.$

$f'(0) = e^0 = 1$

$e^{0.5} = f(0.5) \approx f(0) + f'(0)(0.5 - 0)$

$= 1 + 1 \cdot (0.5) = 1.5$

2. How does the approximation found in Question 1 relate to the exact value of $e^{0.5}$?

   ~~(a) It is an overestimate.~~

   (b) It is an underestimate.

   (c) It is not possible to answer this question without more information.

   Explain why.

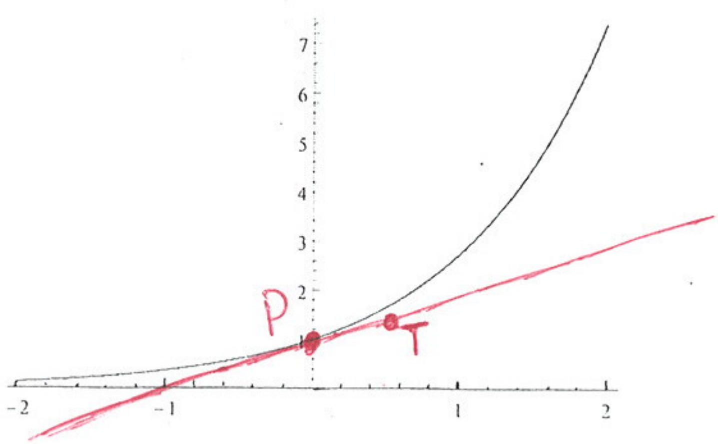
   Graph is concave up, so tangent line is beneath the actual graph.

3. The diagram below shows the graph of the function $f(x) = e^x$ near the origin. Illustrate the linear approximation you did in Question 1 by drawing a *clear* sketch on the diagram below. A clear sketch includes:

   i) the graph of the linear approximation function used in Question 1;

   ii) the point you used to construct such linear approximation function, label this point as P;

   iii) the point you used to estimate $e^{0.5}$, label this point as T if you think it is distinct from point P.



4. Consider the approximation done in Question 1. How big could the error in that approximation be? In other words, find a bound for the error, or equivalently, determine a worst-case error estimate. Explain your answer.

$$|\text{error}| \leq \frac{M}{2}(x-a)^2$$

Here, $x=0.5$, $a=0$, $M \geq \max\limits_{0 \leq x \leq 0.5} |f''(x)|$

$f'(x) = e^x$

$f''(x) = e^x$

note $e \approx 2.72 < 3$.

$|f''(x)| = e^x$ since e^x always positive;

Increasing from 0 to 0.5, so max is at $x=0.5$

$$|f''(x)| \leq e^{0.5}$$

two example conclusions

$e^{0.5} = \sqrt{e}$

$|f''(x)| \leq e^{0.5} < e < 3$

use $M=3$

$$|\text{error}| \leq \frac{3}{2}(0.5-0)^2$$

$|f''(x)| \leq e^{0.5} = \sqrt{e} < \sqrt{3} < 2$

use $M=2$.

$$|\text{error}| \leq \frac{2}{2}(0.5-0)^2$$

Both are valid. Smaller is better.