Limits and Continuity Worksheet

Solutions + CQs.

Tuesday, September 18, 2012

Names and student numbers for group (minimum of 2):

1. ______________________________

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2. ______________________________

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3. ______________________________

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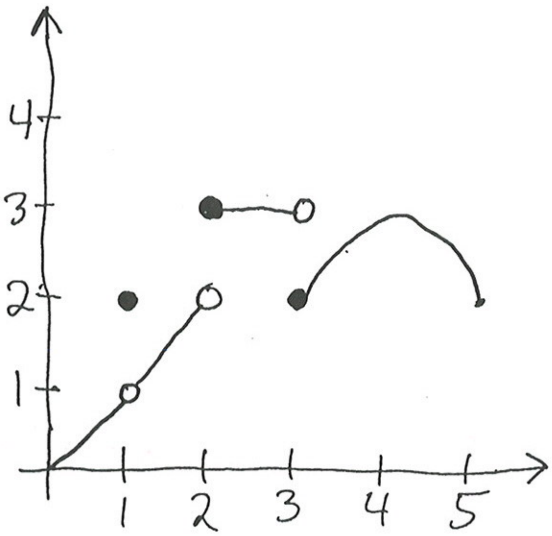
HW: look at Paint Store + web site applet for Thurs.

1. Use the graph of g(x) given in the figure to find the following values, if they exist. If a limit does not exist, explain why.



i. $\lim_{x\to 1} g[x] = 1$ even though $g(1)=2$

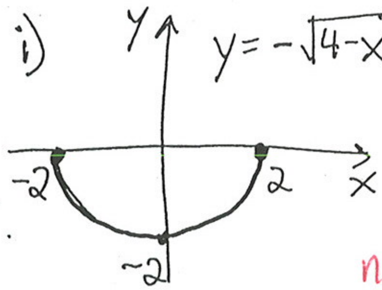
ii. $\lim_{x\to 2^-} g[x] = 2$

iii. $\lim_{x\to 3} g[x]$ DNE $\lim_{x\to 3^-} g(x) = 3$ but $\lim_{x\to 3^+} g(x) = 2$.

iv. $\lim_{x\to 4} g[x] = 3$, continuous at $x=4$.

2. For each of the graphs drawn below, determine where the function is continuous by using interval notation. If it is not continuous somewhere, give a reason why (try using limit notation for this).

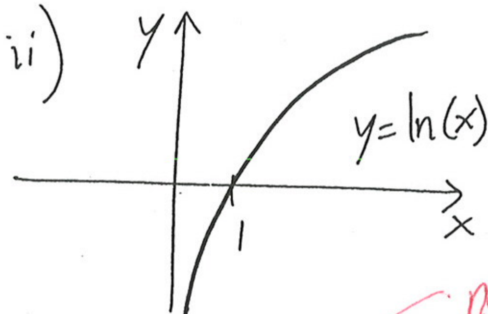
i) $y = -\sqrt{4-x^2}$



points: $\boxed{[-2,2]}$ not defined for $x>2$ nor $x<-2$

intervals: $[-2,2]$

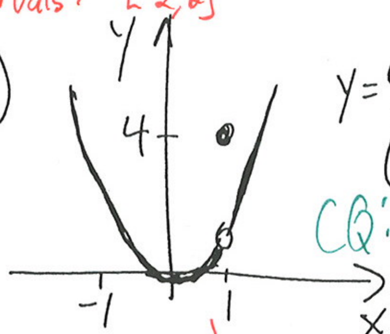
ii) $y=\ln(x)$



points: $\boxed{(0,\infty)}$ not defined for $x \le 0$

intervals: $(0,\infty)$

iii) $y = \begin{cases} x^2 & \text{if } x \ne 1 \\ 4 & \text{if } x = 1 \end{cases}$



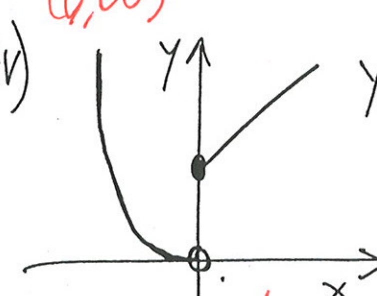
CQ: cts if we replace 4 with ?

A. 0 B. 1 C. -1 D. 3 E. -3.

points: $\boxed{(-\infty,1) \text{ and } (1,\infty)}$ or $x \ne 1$

intervals: $(-\infty,1)$, $(1,\infty)$.

iv) $y = \begin{cases} x^2 & \text{if } x<0 \\ x+1 & \text{if } x \ge 0 \end{cases}$



points: $\boxed{(-\infty,0) \text{ and } (0,\infty)}$ or $x \ne 0$

not continuous at $x=0$

intervals: $(-\infty,0)$ and $[0,\infty)$

CQ: Replace $x+1$ in 2, iv) with $x+c$, continuous for which value of c?

A. 0 B. 1 C. -1 D. 2 E. -2.

3. Is the following function f(x) continuous for all real values of x (looking not just at x=2)? For full credit, you must clearly justify your answer.

$$f(x) = \begin{cases} \frac{x^2-4}{x-2} & \text{if } x \neq 2 \\ 0 & \text{if } x = 2 \end{cases}$$

$$\lim_{x\to 2} f(x) = \lim_{x\to 2} \left(\frac{x^2-4}{x-2}\right)$$

$$= \lim_{x\to 2} \frac{(x+2)\cancel{(x-2)}}{\cancel{(x-2)}}$$

can cancel because $x \to 2$ means x values near 2 but not exactly 2.

$$= \lim_{x\to 2} (x+2)$$

$$= 2+2$$

$x+2$ is continuous, so $\lim_{x\to a}(x+2) = a+2$ for any a.

$$= 4$$

BUT $f(2) = 0$ as defined above.

so $\lim_{x\to 2} f(x) \neq f(2)$

this is 4   this is 0.

hence f is not continuous at $x=2$.

f is continuous for all real values except $x=2$ because $\frac{x^2-4}{x-2} = x+2$ when $x \neq 2$.

OR ~~[illegible]~~ because $\frac{x^2-4}{x-2}$ is rational with $x-2 \neq 0$ for all $x \neq 2$.