Functions and inverse functions

Tuesday, September 11, 2012

Functions

A function is a rule or "map" assigning an output value to an input value. The set (meaning the group) of input values is the domain, while the set of output values is the range.

General notation:

$$f(x) = \text{some expression (usually) involving } x$$

Examples:

$f(x) = 2$ constant, all outputs are 2, no matter what input.

$f(x) = x^2 + 4$

$f(x) = mx + b$

we call m and b parameters, and treat them like fixed numbers.

How familiar is this page about functions?

A. Forgotten everything

B. Remember words, not really the meanings.

C. ~~Domain? Range?~~ Confused by domain range parameters.

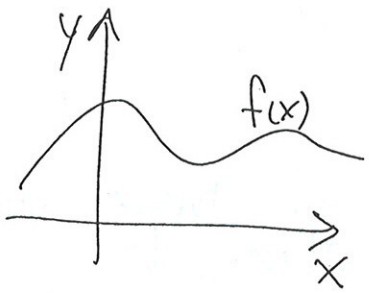
D. All but parameters is okay.

E. I am really good at this.

Graph of a function

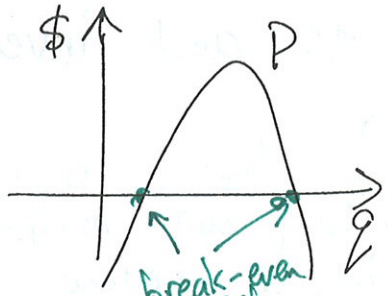
plot $y = f(x)$

"y as a function of x"



plot profit $P(q)$

plot profit $P(q)$
"P as a function of q"



Clicker Q.
Out of i, ii, iii, iv,
How many are graphs of fcn?
A. 0 C. 2 E. 4.
B. 1 D. 3
Which?

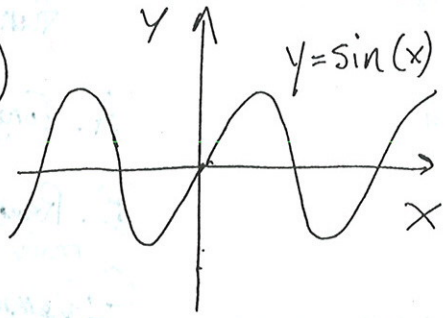
Graphs

To be the graph of a function, the graph must satisfy the vertical line test.

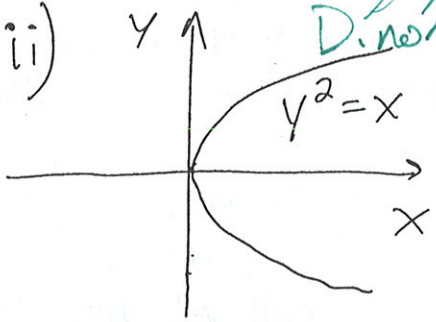
Clicker Q
A. i only
B. i, ii, iii only
C. i, iii only
D. none of them
E. I lost.

Which of these is the graph of a function? Why or why not?

i) ✓

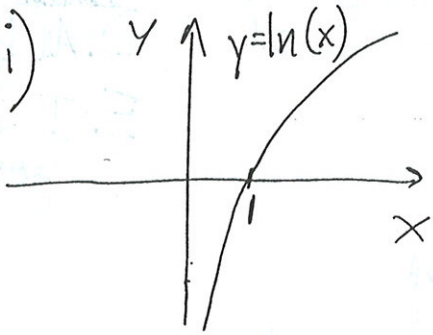


ii)



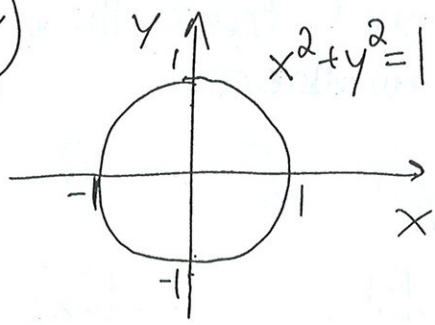
not 1-1 function (not a function).

iii) ✓



also 1-1

iv)



All of these are graphs in the xy plane, but ii and iv have more than one "output" for some x inputs.

One-to-one
1-1

Graph of a 1-1 ("one-to-one") function must satisfy the horizontal line test.

Which of the above are graphs of 1-1 functions? How many?

Which could be the inverse fcn of a fcn? How many?

Inverse Functions

How are the two line tests and inverse functions related?

If graph of function passes horz. line test, then graph of inverse ~~eqn~~ must pass vert. line test.

Our favourite inverse functions in Math 104:

e^x and $\ln(x)$	p and q
We "need" $\ln(x)$ to solve equations like:	It is useful to switch between dependence on price and dependence

$3 = e^{t/12}$ for t

$\ln(3) = \ln(e^{t/12})$

$\ln(3) = \frac{t}{12}\ln(e)$

$t = 12\ln(3)$

and dependence on quantity

Related by the demand equation (or demand relationship)

Example: $p = -\frac{1}{50}q + 300$

This is "p written as a function of q". Can we invert it to have q written as a function of p?

ie. $p = f(q)$

ie. $q = g(p)$

$p - 300 = -\frac{1}{50}q$

$p = f(q) = -\frac{1}{50}q + 300$

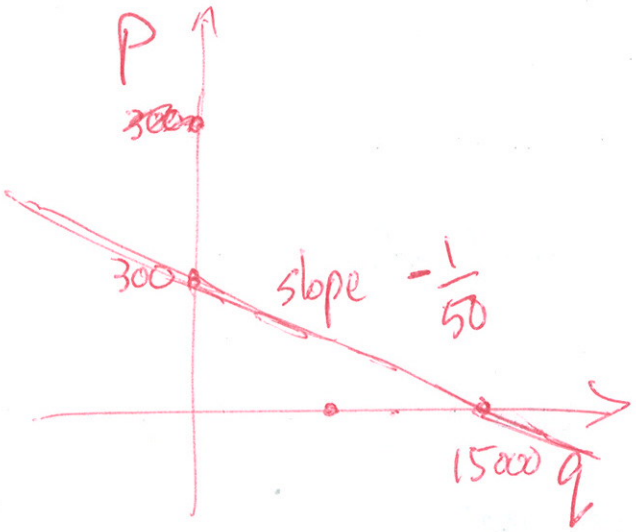
↑ a function of q.

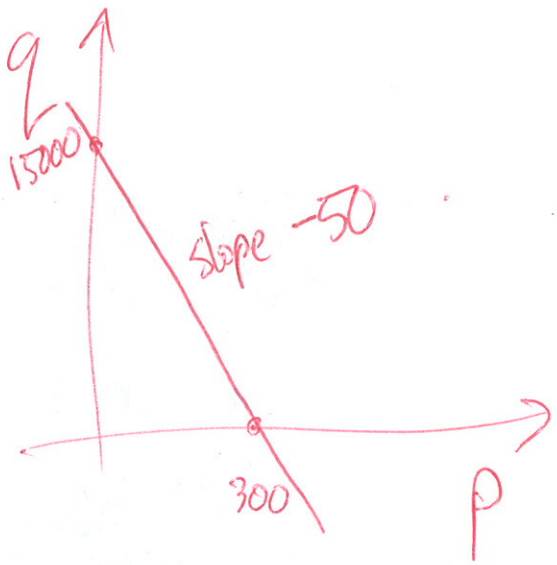
$-50p + 50 \cdot 300 = q$

$g(p)$, a function of p.

$q = -50p + 15000$

$q = g(p) = -50p + 15000$





Note: we do not "swap" p with q in the equation because we swap the axes instead (compare with $f(x)$, $f^{-1}(x)$)