Read the following problem and answer the questions below.

Problem: "Gravel is being dumped from a conveyor belt at a rate of 1.5 m^3/min, and its coarseness is such that it forms a pile in the shape of a cone whose base diameter and height are always equal. How fast is the height of the pile increasing when the pile is 2 m high?"

Note: The volume V of a cone of base radius r and height h is $V = \frac{1}{3}\pi r^2 h$.

1. What are the relevant variables in this problem? Assign names to variables as you see necessary, draw a sketch of the situation described in the problem, and indicate which variables, if any, take on constant values.

$h = 2r$ always.

Variables:
h = height of pile
r = radius of pile base
V = volume of gravel in pile
t = time (minutes).

2. What do you need to find in order to answer the question in this problem? In other words, what quantity is the problem asking for? Express your answer in your own words or in a mathematical notation.

$h'(t_0)$, height of pile when $h = 2$
(OR $\frac{dh}{dt}$ when $h = 2$)

3. Assuming V, r, and h are, respectively, volume, base radius, and height of the pile, and t is time, then the rate at which gravel is added to the pile is
   (a) $\frac{dV}{dr}$
   (b) $\frac{dV}{dh}$
   (c) $\frac{dh}{dt}$
   (d) $\frac{dV}{dt}$

4. Write down an equation that describes how the rate identified in Question 3 relates to the quantity you described in Question 2.

$V = \frac{\pi}{3} r^2 h$ and $h = 2r$, so

$V(t) = \frac{\pi}{3}\left(\frac{h(t)}{2}\right)^2 h(t) = \frac{\pi}{12} h^3(t)$

$V'(t) = \frac{\pi}{12} \cdot 3 (h(t))^2 h'(t)$

5. Find how fast the height of the pile is increasing when the pile is 2 m high. If you need more room, please use the back of the page.

Solve for $h'(t_0)$, using $h(t_0) = 2$, $V'(t_0) = 1.5$:

$V'(t_0) = \frac{\pi}{4}(h(t_0))^2 h'(t_0)$

$1.5 = \frac{\pi}{4} \cdot 2^2 h'(t_0)$ so $h'(t_0) = \frac{1.5}{\pi}$ (m/min)