Teaching methods comparison in a large introductory calculus class

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Introduction

Our work is motivated by a demand for empirical study of less traditional but evidence-based instructional methods for introductory calculus at the undergraduate level (Speer, Smith III, & Horvath, 2010). We gleaned structural ideas from the Physics Education Research community, especially the recent study of Deslauriers, Schewel, & Wieman (2011) involving a week-long intervention by an alternate instructor, though instructional decisions in our study were based on research on student learning in mathematics, with an attempt to situate our analysis in the Action Process Object Schema (APOS) framework. At the research-focused institution in question, the calculus course under study has a population with a range of backgrounds consisting primarily of Commerce and Economics students. Though some applications are tied to these disciplines, much of the syllabus is in common with Calculus 1 elsewhere in North America. Traditional lecture remains the (nearly) uniform choice for instruction, though some local pressure to examine teaching methods has recently arisen. Our research questions are not unlike those of Deslauriers et al. (2011), though we have enhanced the experimental design to improve validity:

Q1: Compared to more traditional lecture-based instruction, will students demonstrate more sophisticated reasoning on an immediate test of learning when high-engagement instruction is implemented for a single topic (100-150 minutes of class time)?

Q2: Will any effects persist to later, standard tests of learning in the course?

Instructional methods

Lesson-structured ideas borrowed from Peer Instruction (Gosch & Mazur, 2001), the Activities, Exercises (ACE) cycle from the Mathematics Education Research community (Weller et al., 2003) and general principles about learning that are now available (Brownell, Brown, & Cocking, 2003) but not known to many university mathematics faculty, particularly at research-focused institutions. Specifically, the goal was to promote “active learning” as described in the science education literature; much of the evidence arises from the K-12 setting though there has been some study at the post-secondary level (Hake, 1998, Michael, 2008).

**Standard week:** Lecture with questions
- *Hands-on lecture* 
- *Clicker questions (1-2 hour)*
- *Whole-class discussions led by instructor*

**Intervention week:** Higher engagement
- *Pre-class assignment*
- In class:
  - Structured handout
  - More clicker questions (5-8 hours)
  - Small group tasks

Experimental Design

1st Term, 1st Year Course where 95% of students have previously taken calculus.
Two comparable (by diagnostics) sections, 150 and 200 students, taught by instructors that are good examples of lecture-based classroom.

Junior instructor trained in research-based methods takes over for one topic (100-150 minutes of in-class time) in each section.

Main conclusions

We observed better performance on conceptual components of the related rates assessments, and a much larger number of students were able to demonstrate the correct picture for linear approximation; the higher-engagement section was stronger in this case. Performance in both sections was very close for computational items and concepts more strongly tied to earlier parts of the course. The data from the final exam was only somewhat supportive of our second research question; some of the gap in performance on the radius in the tank problems persisted, and students in the higher-engagement section were more likely to connect the second derivative to an error bound in the problem.

Related Rates Quiz (QRR)

For a given conical pile of gravel with height equal to base diameter, draw and label a diagram then determine the rate of change of height in time given the volume increase rate and initial size. In Section X (N=177), more students made progress in producing a sensibly-labeled diagram (75% versus 60%, p<0.02) and in solving the problem (graph at right, p<0.02). Unlike Section X, Section A (N=131) had seen a complete worked example during class with the “cone geometry” for an inverted conical tank draining, and were more likely to use the standard proportionality relationship for the radius and height, but were still not as successful in their solutions.

Related Rates Midterm Question (MTQR)

Compute the rate of change of height for each of an inverted cone and cylinder-shaped water tank of the same height and volume, given the same volume fill rate and initial water depth.

Related Rates Quiz (QLR)

Estimate \( e^{0.5} \) using linear approximation with the tangent line at \( x=0 \). Determine if it is an over- or underestimate, and draw the situation on the provided graph of \( e^x \).

Linear Approximation Quiz (QLA)

Here, N=106 for Section X and N=133 for Section A. Successful use of the linear approximation formula \( f(x) = f(a) + f'(a)\Delta x \) was similar in both sections, with 52% in Section X and 60% in Section A comparing the estimate correctly (p>0.2). Students in Section X were less likely to declare their estimate an underestimate (66% versus 82% for Section A, p<0.01), but were much more successful in drawing the correct tangent line; 29% of the Section A students drew the linearization on 0.5 (5.5m on right, the ‘tan’ (‘x’ error) instead of at 0 (on left) despite having seen multiple correct diagrams presented during class.

Final Exam (FE)

Performance on the final exam is still under review, but the “gap” in working with tank radius appears a bit smaller on the final exam, while analysis of the error in linear approximation favors the higher-engagement section (computational items were still similar between sections).

References


Further information:

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