Today: review the main properties of lines and parabolas.

**Lines**

- Constant slope represents direction and steepness.

Eq. of a line: \( y = mx + b \)

\[
\frac{\text{rise}}{\text{run}} = \text{slope}
\]
How to find the eq. of a line

need slope \( m \)

at least one point on the line \((x_0, y_0)\)

\[
y = mx + b
\]

\((x_0, y_0)\) lies on line, hence

\[
y_0 = mx_0 + b
\]

\[
b = y_0 - mx_0
\]

so

\[
y = mx + y_0 - mx_0
\]

\[
y = y_0 + m(x-x_0)
\]

point-slope form
Problem: Find the eq. of the line that goes through (3,4) and (-1/2,1).

* find slope between (3,4) and (-1/2,1)

\[
m = \frac{\text{rise}}{\text{run}} = \frac{4-1}{3-(-1/2)} = \frac{3}{3+1/2} = \frac{6}{7}
\]

* pick a point (3,4)

* use point-slope form

\[
y = 4 + \frac{6}{7}(x-3)
\]
Functions

What is a function?

Defn: a function is a relationship/rule (domain) between two variables.

A function is a rule for assigning to each element of a set \( D \) a single element of a set \( R \).

\[ y = f(x) \]
How to represent a function?
- graph
- table of values
- equation

\[ y = 3x^2 \]

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
</tr>
<tr>
<td>-1</td>
<td>3</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Linear functions
\[ y = \boxed{m}x + b \]
constant
Other properties of lines

* Two lines with equal slope are called parallel \( m_1 = m_2 \)

\[
\begin{align*}
2x + 4y &= 10 \\
m_1 &= -\frac{b}{a} = -\frac{2}{4} \\
y &= -\frac{1}{2}x + 1 \\
m_2 &= -\frac{1}{2}
\end{align*}
\]

* Two lines with slope \( m_1 \) and \( m_2 \) such that \( m_2 = -\frac{1}{m_1} \) are called perpendicular

\[
\begin{align*}
y &= -\frac{1}{2}x + 1 \\
m_1 &= -\frac{1}{2} \\
y &= 2x + 3 \\
m_2 &= +2
\end{align*}
\]
Quadratic functions

\[ y = x^2 + 4x - 3 \]

\[ y = x^2 \]

Graph is called \textit{parabola}.

General form \[ y = ax^2 + bx + c \]

\textbf{y-intercept.} is found by \( x = 0 \)

\[ y = a \cdot 0 + b \cdot 0 + c \]

\[ y = c \]

\textbf{x-intercept(s)} is found by \( y = 0 \)

\[ ax^2 + bx + c = 0 \]

\( \textcircled{a} \) Try by factoring.
or use quadratic formula
\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

Ex: \( y = x^2 - 4x + 3 \) graph this parabola.

find y-intercept: \( y = 3, x = 0 \)

x-intercepts: try to factor:
\[ x^2 - 4x + 3 = 0 \]
\[ (x - 3)(x - 1) = 0 \]  
\( x = 3, y = 0 \)  
\( x = 1, y = 0 \)

In general, \( y = ax^2 + bx + c \)

if \( a > 0 \), \( \cup \) "opens up"  
\( a < 0 \), \( \cap \) "opens down"

back to ④ \[ y = x^2 - 4x + 3 \]  
\( a = 1 > 0 \)  \( \cup \)
\[ y = -3x^2 + 4x - 1 \] sketch graph.

\[ y \text{-intercept } y = -1 \quad (0, -1) \]

\[ x \text{-intercepts} \quad -3x^2 + 4x - 1 = 0 \quad a = -3 \quad b = 4 \quad c = -1 \]

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

\[ = \frac{-4 \pm \sqrt{16 - 12}}{-6} \]

\[ = \frac{-4 \pm 2}{-6} \]

\[ \frac{-4 + 2}{-6} = \frac{1}{3} \]

\[ \frac{-4 - 2}{-6} = 1 \]

"opens down" \[ a = -3 \]
**Problem:** Find the value of the constant \( k \) such that \( y = 2x^2 + k \) goes through \( (\frac{1}{2}, 4) \).

\[
y = 2x^2 + k \text{ goes through } (\frac{1}{2}, 4)
\]

So, \( 4 = 2 \cdot (\frac{1}{2})^2 + k \) solve for \( k \).

\[
4 = 2 \cdot \frac{1}{4} + k
\]

\[
k = 4 - \frac{1}{2} = \frac{7}{2}
\]
Piecewise Functions (or multilime functions)

A function defined by multiple subfunctions, each subfunction applying to a certain interval of the function's domain.

Example:

\[ f(x) = \begin{cases} 
  \frac{1}{2}x + 3 & x \leq 2 \\
  -x^2 + 7x - 6 & x > 2 
\end{cases} \]

Domain: all \( x \) in \( \mathbb{R} \).

Sketch:
if \( x < 2 \), \( y = -\frac{1}{2}x + 3 \)

y-intercept: \( x = 0 \)
\( y = 3 \)

slope \( m = \frac{x}{2} - \frac{1}{2} \)

if \( x = 2 \), \( f(2) = -\frac{1}{2}(2) + 3 = 2 \)

if \( x > 2 \), \( y = -x^2 + 7x - 6 \)

opens down, \( a = -1 \)

y-intercept \( y = -6 \) not included

x-intercepts \(-x^2 + 7x - 6 = 0 \)
\( x^2 - 7x + 6 = 0 \)
\( (x-6)(x-1) = 0 \)

\( x = 6, x = 1 \) not included

near \( x = 2 \), \( y = -(2)^2 + 7(+2) - 6 \)
\( = -4 + 14 - 6 = 4 \)

\( x^2 - 2 \) \( y = -(-2)^2 \) ....
Absolute-valued functions

\[ y = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases} \]

| \( x \) | \(|x|\) |
|---|---|
| 1 | 1 |
| -1 | \(|-1| = 1 |
| -\frac{1}{2} | \(|-\frac{1}{2}| = \frac{1}{2} |
| -100 | \(|-100| = 100 |

\[ y = |x+3| = \begin{cases} x+3 & \text{if } x+3 \geq 0 \\ -(x+3) & \text{if } x+3 < 0 \end{cases} \]

which is equivalent to

\[ y = |x+3| = \begin{cases} x+3 & \text{if } x \geq -3 \\ -x-3 & \text{if } x < -3 \end{cases} \]

| \( x \) | \(|x+3|\) |
|---|---|
| 1 | \(|1+3| = 4 |
| -1 | \(|-1+3| = 2 |
| -2 | \(|-2+3| = 1 |
| -3 | \(|-3+3| = 0 |
| -4 | \(|-4+3| = 1 |
| -5 | \(|-5+3| = 2 |
Rational Functions

ex: \( y = \frac{x+1}{x-3} \)

in general, \( f(x) = \frac{N(x)}{D(x)} \)

where \( N(x) \) and \( D(x) \) are polynomials.

(recall: polynomial is a sum of powers of \( x \).
\[ \rightarrow x^3 - x^2 + \frac{1}{2}x + 10 \]
\[ x^{10} + \frac{1}{2}x^2 - x \]
\[ x^{10} \]

Domain of \( y = \frac{D(x)}{x-3} \) all \( x \neq 3 \)

In general, the domain of \( y = \frac{N(x)}{D(x)} \) is all \( x \) that make \( D(x) \neq 0 \).
\[ y = \frac{x+2}{x^2-1} \]

Domain: \[ x^2-1 \neq 0 \quad \Rightarrow \quad x \neq \pm 1 \]
\[(x+1)(x-1) \neq 0 \quad \Rightarrow \quad x \neq \pm 1\]

\[ x+1 \neq 0 \quad \Rightarrow \quad x \neq -1 \]

\[ x-1 \neq 0 \quad \Rightarrow \quad x \neq 1 \]

\[ y = \frac{x^2-1}{x^3-8}, \quad x^3 - 8, \neq 0 \]

\[ x \neq 8 \]
\[ x \neq 3^{\frac{3}{2}} \neq 2 \]$^2$ \[ (x - 2)^3 = -8 \]

Domain: all $x \neq 2$
Function Composition

how to create new functions from old functions.

ex: \( y = |x| \leftarrow \text{output} \)

\[
\begin{align*}
y &= |x+3| \leftarrow \text{output} \\
\end{align*}
\]

input

\[
\begin{align*}
z &= (x+3) \leftarrow \text{output} \\
\end{align*}
\]

input

this is an example of function composition.

\[
\begin{align*}
g(x) &= x+3 \\
f(z) &= |z| \\
\text{let } z &= g(x), \text{ then } f(g(x)) &= |x+3|.
\end{align*}
\]