1. In this question you will be given a problem and an incomplete solution to the problem. The solution contains some reflection questions and some missing steps. Your task is to answer the reflection questions and complete the calculations.

**Problem:** Show that the curve $y = x^2$ intersects the curve $y = 1/\sqrt{x}$ at a point where the tangent lines to each curve are perpendicular to each other.

**Solution:**

$$x^2 = \frac{1}{\sqrt{x}} \quad 1. \text{How did we get this equation?} \quad 2. \text{Why did we set it up?}$$

$$x^4 = \frac{1}{x} \quad 3. \text{How did we get this?}$$

$$x^5 = 1 \quad 4. \text{What does } x \text{ represent here?}$$

For the curve $y = x^2$,

$$y' = 2x, \quad 5. \text{Why did we compute } y' \text{ here?}$$

and

$$y'(1) = 2 \quad 6. \text{Why did we evaluate } y' \text{ at } x = 1?$$

For the curve $y = 1/\sqrt{x}$,

$$y' = \ldots, \quad 7. \text{Complete the calculation}$$

and

$$y'(1) = \ldots \quad 8. \text{Complete the calculation}$$

We conclude that the tangent lines are perpendicular because ... 9. Complete the sentence.

2. Reflect on the solution to the previous problem and the key points highlighted by the reflection questions. Then read the following problem and follow the steps below to solve it.

**Problem** A tangent line is drawn to the curve $y = 1/x$ at a point $P$. Find the area of the triangle formed by the tangent line and the coordinate axes and explain why the above triangle has always the same area, no matter where $P$ is located on the curve.

(a) Let $p$ be the $x$-coordinate of $P$. Find the $y$-coordinate of $P$.

(b) Find the equation of the tangent line at $P$, in terms of $p$.

(c) Find the area of the triangle formed by the tangent line above and the coordinate axes.

(d) Explain why the above triangle has always the same area, no matter where $P$ is located on the curve.

(e) Finally, draw a sketch of the situation.
3. For each one of the following functions, compute the derivative in at least two different ways. For example, a quotient of functions can be differentiated by applying the quotient rule; alternatively, sometimes it is possible to simplify the quotient first and obtain a sum of powers, which can be differentiated by the power and sum rules.

(a) \( y = \frac{x^4 - 3x^{-1} + 5\sqrt{x}}{x^2} \)

(b) \( f(t) = (\sqrt{t} - t^2)(5e^x + \frac{2}{3}x) \)