1. \( y = x^2 + x - 2 \)

Find intercepts.

- \( x = 0, y = -2 \)
- \( y = 0, x^2 + x - 2 = 0 \)
  \( (x-1)(x+2) = 0 \)
  \( x = 1, x = -2 \)

The secant line through \((-1, -2)\) and \((x, y)\) has slope

\[
m = \frac{y - (-2)}{x - (-1)} = \frac{y + 2}{x + 1}
\]

If \( x = -0.98 \), then
\[
m = \frac{(-0.98)^2 + (-0.98) - 2 + 2}{-0.98 + 1} = \frac{-0.0196}{0.02} = -0.98
\]

If \( x = -1.03 \), then
\[
m = \frac{(-1.03)^2 + (-1.03) - 2 + 2}{-1.03 + 1} = \frac{0.0309}{-0.03} = -1.03
\]

If \( x = -1 + h \), then
\[
m = \frac{(-1+h)^2 + (-1+h) - 2 + 2}{(-1+h) + 1} = \frac{1 - 2h + h^2 - 1 + h}{h} = \frac{h^2 - h}{h} = \frac{h(h-1)}{h}
\]

When \( h \) is very small (close to 0), I except the slope of the secant to approach -1. This can be verified by simplifying \( \frac{h(h-1)}{h} = h-1 \), which is allowed because \( h \neq 0 \) (\( h \) cannot be 0 because otherwise the second point would coincide with the first point and we would not be able to find the slope of the line).
(a) \( g(t) = \begin{cases} 
1 & \text{if } t \leq 0 \\
t + 1 & \text{if } 0 < t < 2 \\
t^2 - 1 & \text{if } t \geq 2
\end{cases} \)

(b) \( f(x) = x + 3|x| = \begin{cases} 
x + 3x = 4x & \text{if } x > 0 \\
x - 3x = -2x & \text{if } x < 0
\end{cases} \)

(c) To estimate the slope of the tangent line at \((3, g(3))\), consider a nearby point on the curve, e.g. \((3.1, g(3.1))\) and compute the slope of the secant line:

\[
m = \frac{g(3.1) - g(3)}{3.1 - 3} = \frac{(3.1)^2 - 1 - 8}{0.1} = \frac{9.61 - 9}{0.1} = \frac{0.61}{0.1} = 6.1
\]

The slope of the tangent line at \((3, g(3))\) is about 6.1.
(3) \[ f(x) = 3x^2 - x + 2 \]. Then,

1. \( f(2) = 3 \cdot (2)^2 - 2 + 2 = 12 \)
2. \( f(a^2) = 3(a^2)^2 - (a^2) + 2 = 3a^4 - a^2 + 2 \)
3. \[ [f(a)]^2 = (3a^2 - a + 2)^2 = 9a^4 - 6a^3 + 13a^2 - 4a + 4 \]
4. \( f(a+1) = 3(a+1)^2 - (a+1) + 2 = 3a^2 + 5a + 4 \)

5. \( \frac{f(x+h) - f(x)}{h} = \frac{3(x+h)^2 - (x+h) + 2 - (3x^2 - x + 2)}{h} = \frac{3x^2 + 6xh + 3h^2 - x - h + 2 - 3x^2 + x - 2}{h} = \frac{3h^2 + h(6x-1)}{h} = \frac{h(3h + 6x - 1)}{h} = 3h + 6x - 1 \)
4. \( Y \) (ft) \( t \) (s)

(a) avg. velocity over \([0,30]\) = \(\frac{300-0}{30-0}\) = 10 ft/s

(b) avg. velocity over \([10,30]\) = \(\frac{300-100}{30-10}\) = 10 ft/s

(c) We can estimate the instantaneous velocity of the car at \(t=10\) s by estimating the slope of the tangent line to the graph at that point. The tangent goes through the points \((5,25)\) and \((15,175)\) so its slope is \(\frac{175-25}{15-5} = 15\).

Hence we estimate the velocity of the car at \(t=10\) s to be about 15 ft/s.

At \(t=20\) s the tangent to the graph appears to be horizontal so its slope is 0. The car has zero velocity.

At \(t=30\) s, the tangent line is very steep. Its slope is approximately \(\frac{300-0}{30-27}\), so we estimate the velocity of the car to be about 100 ft/s.
(d) Between $t=15$ and $t=20$, the position of the car is constant, which means the car is not moving.

(e) The negative velocity at $t=25$ means the car has reversed its direction of motion and it is now moving towards the origin, i.e., the starting point.

5. $f(x)$ must satisfy:
   
   $f(0) = \frac{1}{2}$
   $f(1) = 2$
   $f^{-1}$ exists and has domain $x > \frac{1}{2}$

   An example of one such function is

   ![Graph of a function]

   This function has domain $x > 0$