MATH 110 April Exam, April 18th, 2016

Duration: 150 minutes

*This test has **10 questions** on **12 pages**, for a total of 100 points.*

- Read all the questions carefully before starting to work.
- All questions require a full solution; you should give complete arguments and explanations for all your calculations; answers without justifications will not be marked.
- Continue on the back of the previous page if you run out of space.
- Attempt to answer all questions for partial credit.
- This is a closed-book examination. **None of the following are allowed**: documents, cheat sheets or electronic devices of any kind (including calculators, cell phones, etc.)

First Name: __________ Last Name: __________

Student-No: __________ Section: __________

Signature: __________

Question:	1	2	3	4	5	6	7	8	9	10	Total
Points:	10	14	7	8	15	12	9	9	8	8	100
Score:											

Student Conduct during Examinations

1. Each examination candidate must be prepared to produce, upon the request of the invigilator or examiner, his or her UBCcard for identification.

2. Examination candidates are not permitted to ask questions of the examiners or invigilators, except in cases of supposed errors or ambiguities in examination questions, illegible or missing material, or the like.

3. No examination candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination. Should the examination run forty-five (45) minutes or less, no examination candidate shall be permitted to enter the examination room once the examination has begun.

4. Examination candidates must conduct themselves honestly and in accordance with established rules for a given examination, which will be articulated by the examiner or invigilator prior to the examination commencing. Should dishonest behaviour be observed by the examiner(s) or invigilator(s), pleas of accident or forgetfulness shall not be received.

5. Examination candidates suspected of any of the following, or any other similar practices, may be immediately dismissed from the examination by the examiner/invigilator, and may be subject to disciplinary action:

   (i) speaking or communicating with other examination candidates, unless otherwise authorized;

   (ii) purposely exposing written papers to the view of other examination candidates or imaging devices;

   (iii) purposely viewing the written papers of other examination candidates;

   (iv) using or having visible at the place of writing any books, papers or other memory aid devices other than those authorized by the examiner(s); and,

   (v) using or operating electronic devices including but not limited to telephones, calculators, computers, or similar devices other than those authorized by the examiner(s)(electronic devices other than those authorized by the examiner(s) must be completely powered down if present at the place of writing).

6. Examination candidates must not destroy or damage any examination material, must hand in all examination papers, and must not take any examination material from the examination room without permission of the examiner or invigilator.

7. Notwithstanding the above, for any mode of examination that does not fall into the traditional, paper-based method, examination candidates shall adhere to any special rules for conduct as established and articulated by the examiner.

8. Examination candidates must follow any additional examination rules or directions communicated by the examiner(s) or invigilator(s).

Full-Solution Problems. In the following questions, justify your answers and **show all your work**. Unless otherwise indicated, **simplification of answers are required**.

10 marks

1. (a) Find the x- and y-intercepts of the line that is tangent to the curve $y = \sqrt{x}$ at the point where $x = 2$.

   (b) If a rock is thrown upward on the planet Mars with a velocity of 10 m/s, its height (in metres) above the ground level after t seconds is given by $h(t) = 10t - 2t^2$. Find the velocity of the rock when it hits the surface.

   (c) Let $h(x) = \sin^2(2\pi x)$. Find $h'(x)$.

   (d) Express the derivative of $f(x) = \dfrac{1}{x^2 + 1}$ at $x = 2$ as a limit. If necessary, simplify your answer so that it contains only the variable needed to evaluate the limit. You do not need to evaluate the limit.

14 marks

2. (a) For the curve defined by the equation $\sqrt{xy} = x^2y - 2$, find the slope of the tangent line at the point $(1, 4)$.

(b) Find the coordinates of the inflection point of the function $g(x) = xe^x$.

(c) Find the absolute minimum of $f(x) = \ln(x^2 + x + 1)$ on the interval $[-1, 1]$. Make sure you justify your answer.

(d) Find an antiderivative of $\sin x + \dfrac{2}{\sqrt{x}}$.

7 marks

3. Consider the following function, where k is a constant,

$$f(x) = \begin{cases} e^x & \text{if } x \leq 0 \\ k(x-1)+2 & \text{if } 0 < x < 1 \\ x^2+1 & \text{if } x \geq 1. \end{cases}$$

(a) For $k = 0$, which one of the following statements is the best reason why $f(x)$ is **not continuous** at $x = 0$?

(i) $f(0)$ does not exist.

(ii) $\lim_{x \to 0} f(x)$ does not exist.

(iii) Both (i) and (ii).

(iv) $f(0)$ and $\lim_{x \to 0} f(x)$ exist, but they are not equal.

(b) For $k = 2$, which one of the following statements is the best reason why $f(x)$ is **continuous** at $x = 1$?

(i) $f(1)$ exists.

(ii) $\lim_{x \to 1} f(x)$ exists.

(iii) Both (i) and (ii).

(iv) $f(1)$ and $\lim_{x \to 1} f(x)$ exist, and they are equal.

(c) For what value(s) of k does the equation $f(x) - e^{-(x-1)} = 0$ have at least one solution? Explain why. Also, make sure you state what theorem(s) you may be using to justify your claim and why they apply.

8 marks 4. Evaluate the following limits, if they exist. If a limit does not exist, specify whether it is (positive or negative) infinity or neither.

(a) $\displaystyle\lim_{t\to 3}\frac{t-3}{(t+1)^2}$.

(b) $\displaystyle\lim_{x\to 2}\frac{x^2-3x+2}{x-2}$.

(c) $\displaystyle\lim_{x\to\infty}(x+1)(x+3)e^{-x}$.

(d) $\displaystyle\lim_{x\to 5^-}\frac{x}{(x-5)^2}$.

15 marks 5. Let $f(x) = \dfrac{x-1}{x^2}$.

(a) State the domain of f.

(b) Find the y- and x-intercepts of f, if they exist.

(c) Find all the vertical and horizontal asymptotes of f, if they exist.

(d) Compute $f'(x)$.

(e) Find the intervals where f is increasing and the intervals where it is decreasing.

(f) Find all local maximum and minimum values of f, if they exist.

(g) Compute $f''(x)$.

(h) Find the intervals where f is concave up and the intervals where it is concave down.

(i) Make a large sketch of the graph of the function f. Make sure you identify the x-coordinate of any intercepts as well as local extreme values and inflection points, if they exist.

12 marks

6. A television camera is positioned 1000 m from the base of a rocket launching pad. The angle of elevation of the camera has to change at the correct rate in order to keep the rocket in sight. Also, the mechanism for focusing the camera has to take into account the increasing distance from the camera to the rising rocket. Answer the questions below. Make sure your solution includes a sketch that is labeled with the same variables used in your calculation. Your answer should be a numerical value, but you do not need to simplify it.

   Assume the rocket rises vertically and its speed is 500 m/s when it has risen 2000 m.

   (a) How fast is the distance from the television camera to the rocket changing at that moment?

   (b) If the television camera is always kept aimed at the rocket, how fast is the camera's angle of elevation changing at that same moment?

9 marks

7. If 1200 cm^2 of material is available to make a box with a square base and open top, find the largest possible volume of the box. Justify your answer. Recall the volume V of a rectangular box with width w, length l, and height h is $V = wlh$.

$\boxed{\text{9 marks}}$ 8. (a) Use an appropriate linear approximation to estimate $\dfrac{1}{4.16}$. Simplify your answer.

(b) Is your answer in part (a) an overestimate, an underestimate, or exactly equal to, the actual value of $\dfrac{1}{4.16}$? Justify your answer.

(c) Sketch a graph to illustrate your claim in part (b). Your sketch must show *clearly* the graph of the linear function used to compute the approximation in part (a) and how the approximation compares to the exact value of $\dfrac{1}{4.16}$. Your sketch need not be in scale.

(d) Find the 2nd-degree Taylor Polynomial of $\dfrac{1}{x}$ at 4. You do not need to simplify your answer.

8 marks 9. Suppose that the function f is differentiable for all x and that $f(-1) = -1$ and $f(2) = 5$.

(a) Prove that there is a point on the graph of f at which the tangent line is parallel to the line with the equation $y = 2x$. Make sure you state what theorem(s) you use to support your claim and why they apply.

(b) Suppose also that $f'' < 0$ everywhere. Is it possible that $f'(2) = 3$? Explain why or why not.

$\boxed{\text{8 marks}}$ 10. What is the shortest possible length of the line segment that is cut off by the first quadrant and is tangent to the curve $y = \dfrac{3}{x}$ at some point?