WORKSHOP 1.3
Solutions

Part A

There are two ways to solve this problem, whether the side we consider the length is parallel or perpendicular to the river. I added a figure for the former case.
First we set the dimensions of the rectangle as $x$ for the length and $y$ for the height. We know that the area of the rectangle is expressed as
$$A = xy.$$ However we need an expression of $x$ only. We know that the sum of the lengths of the three sides must be $240$ m. So we have
$$240 \text{ m} = x + 2y.$$ We are required to find a formula of $x$ only, so we may solve for $y$:
$$y = \frac{240 \text{ m} - x}{2}.$$ Finally we can substitute back in the area formula to obtain
$$A(x) = \frac{240 \text{ m}x - x^2}{2}.$$ If we assume that the length is perpendicular to the river we obtain $y = 240 \text{ m} - 2x$ and $A(x) = 240 \text{ m}x - 2x^2$.

Part B

First we draw the diagram of our window.
We note that the window has two main dimension: its width $x$ and the height of its rectangle section $y$. The area is computed by adding the area of the bottom rectangle and that of the half disk on top.
$$A = xy + \frac{\pi r^2}{2}.$$ Once again we need to replace $y$ in that expression as we want a formula involving only $x$. We use the perimeter of $10$ m provided.
$$10 \text{ m} = 2y + x + \frac{\pi x}{2}$$
We can solve for $y$:
$$y = \frac{10 \text{ m} - \left(1 + \frac{\pi}{2}\right)x}{2}.$$
Finally we substitute back into our area formula:

\[ A(x) = \frac{10 \text{ m} \cdot x - (1 + \frac{x}{2}) \left( \frac{x}{2} \right)^2}{2} \]

**Part C**

The first step is to draw the diagram of the poster.

We define the the total width as \( x \) and the total height as \( y \). We know that the total area must be 1000 cm\(^2\):

\[ 1000 \text{ cm}^2 = xy. \quad (1) \]

We then need to compute the printable area. That is the total area minus the area of each corner and margin rectangles:

\[ PA = 1000 \text{ cm}^2 + 4 \cdot 35 \text{ cm}^2 - 2x \cdot 7 \text{ cm} - 2y \cdot 5 \text{ cm}. \]

We add \( 4 \cdot 35 \text{ cm}^2 \) in the formula because we removed each corner twice. We need to find an expression as a function of one side’s length only so we use the total area formula (1) and solve for \( y \):

\[ y = \frac{1000 \text{ cm}^2}{x}. \]

Finally we substitute back into our printable area formula:

\[ PA(x) = 1000 \text{ cm}^2 + 4 \cdot 35 \text{ cm}^2 - 2x \cdot 7 \text{ cm} - 2 \frac{1000 \text{ cm}^2}{x} \cdot 5 \text{ cm}. \]
The domain of our printable formula is the set of all possible widths $x$ that could produce a poster. If we just consider the fact that we need side margins of 5 cm, the minimal width would be 10 cm. For the maximal width we set the height to 14 cm, the minimal height and the width should be $\frac{1000 \text{ cm}^2}{14 \text{ cm}}$. The domain is thus the interval $[10 \text{ cm}, \frac{1000 \text{ cm}^2}{14 \text{ cm}}]$.

For the bonus question. We first need to find a formula for the output per tree as a function of $n$. To do this we find the formula of the line that goes through $(50, 350)$ with a slope of $-10$. We can use the point-slope method to find this:

$$UO = -10(n - 50) + 350.$$  

The total output is simply the output per tree times the total number of trees.

$$TO = -10n(n - 50) + 350n.$$