WORKSHOP 1·13

Handout

Warm-up Problem

Your village depends for its food on a nearby fishpond. One day the wind blows a water lily seed into the pond, and the lily begins to grow. It doubles its size every day, and its growth is such that it will cover the entire pond (and smother the fish population) in 45 days. You are away on vacation, and the villagers depend on your vigilance and intelligence to handle their affairs. Without you present they will take action about that lily only on regular work days, and only when the lily covers half the pond or more. How many days after it first starts growing will the lily cover half the pond? The day it reaches that size happens to be a holiday....what do you do?

Worked Example:

Assume that the number of bacteria at time t (in minutes) follows an exponential growth model $P(t) = Ce^{kt}$, where C and k are constant. Suppose the count in the bacteria culture was 300 after 15 minutes and 1800 after 40 minutes. When will the culture contain 3000 bacteria?

Main Problem

Two cities, Growth and Decay, have populations that are respectively increasing and decreasing, at (different) rates that are proportional to the respective current population (that is, we can model the population size as an exponential function). Growth's population is now 3 million and was 2 million 10 years ago, and Decay's population is now 5 million and was 7 million 10 years ago.

Question: How many years from now will Growth's population equal 4 million?

Question: How many years from now will Decay have half the population of Growth?

Question: At what rate is the population changing for the two towns at that moment?

Bonus Question 1: Find a formula for the rate of change of the combined population of Growth and Decay at time t. Show that the rate goes from negative to positive. What does that mean in terms of population?

Bonus Question 2: A third city, Grecay, has a lot of seasonal workers, such that its peak in the summer (around July 1st) it has the same population as Growth and at its lowest in the winter (around January 1st) it has only half of the population. Using the formula you obtained previously for Growth's population, try to find a possible formula for Grecay's population after t years. Note that the population has to remain continuous.

Hint: Try a population function of the form $P(t) = G(t) \cdot h(t)$, where $G(t)$ is Growth's population and $h(t)$ is a periodic function, like sine or cosine, might be useful here.