

SCIENCE ONE, MATHEMATICS - HOMEWORK #2

Due on Monday October 23 at 10AM.

Hand in a **full solution** to the following problems.

1. In this problem you will use a suitable linear approximation (tangent line approximation) to estimate the coordinate of a point that lies on a curve that is not a function. Do not use a calculator in this question.

Consider the point of y -coordinate $\frac{31}{16}$ that lies on the curve of equation $x^3 + xy^2 + y^3 = 13$. Using a suitable linear approximation, find an estimate of the x -coordinate of the point. Show your work. You may find useful to know that the point $(1, 2)$ lies on the curve.

2. Use Calculus to prove that any tangent line to a circle is perpendicular to the radius drawn to the point of tangency.
3. A large predator fish of (vertical) size S is approaching a small fish at a constant speed v .

(a) Express the rate of change of the visual angle perceived by the prey in terms of the size and speed of the approaching predator, and its distance away from the prey. The visual angle is the angle that an image subtends on the retina of the observer.

(b) Visual angles are important factors in studies of predator avoidance. A model proposed by L. Dill, a behavioural ecologist at Simon Fraser University, about the behaviour of the Zebra Danio (a small tropical fish) suggested that “Zebra Danios react to an approaching predator when the rate of change of the angle subtended by the predator at the prey’s eye exceeds some threshold level” (L. Dill (1974) The escape response of the Zebra Danio (*Brachydanio rerio*). I. The stimulus for escape. *Animal Behaviour* 22, 711-722).

Considering the prey-predator scenario described in this problem and using the results you found in part (a), discuss how the rate of change of the visual angle at the prey’s eye changes as the predator gets closer to the prey. Does the rate increase or decrease?

(c) Suppose an escape reaction is triggered when the rate of change of the visual angle reaches the threshold value r_{crit} . Find the distance x_{react} between prey and predator at which an escape response is triggered.

(d) For a predator of given size S , what values of v would not result in an escape response from the prey?

(e) For a given “hunting” speed v , what values of S would not result in an escape response from the prey?

(f) Using your results from part (d) and (e) explain why large predators or slowly moving predators would have higher success rate at eating the small Zebra Danio.