Problem 1: [1 point] True/False: If $\lambda$ is an eigenvalue of a matrix $A$ then it is always an eigenvalue of $A^T$.
A: False    B: True

Problem 2: [1 point] Consider the non-zero $n \times n$ matrix $A = \begin{pmatrix} 1 & 2 & \cdots & n \\ 1 & 2 & \cdots & n \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 2 & \cdots & n \end{pmatrix}$. How many linearly independent eigenvectors does $A$ have?
A: $n - 1$    B: $n$    C: 2    D: 1    E: 0

Problem 3: [1 point] For the matrix $A$ in the previous question, how many distinct eigenvalues does it have?
A: $n - 1$    B: $n$    C: 2    D: 1    E: 0

Problem 4: [1 point] Calculate the eigenvalues $\lambda_{1,2,3}$ of the matrix $B = \begin{pmatrix} -3 & 2 & -1 \\ 0 & -2 & 1 \\ 0 & 0 & -1 \end{pmatrix}$
A: $\lambda_1 = -3, \lambda_2 = 2, \lambda_3 = -1$    B: $\lambda_1 = 3, \lambda_2 = 2, \lambda_3 = 1$    C: $\lambda_1 = -6, \lambda_2 = -6, \lambda_3 = -6$    D: $\lambda_1 = -3, \lambda_2 = -2, \lambda_3 = -1$    E: $\lambda_1 = 1, \lambda_2 = 4, \lambda_3 = 9$

Problem 5: [2 points] Let $A = \begin{pmatrix} -3 & 2 & a \\ 2 & -2 & b \\ 4 & 2 & 1 \end{pmatrix}$. Find $a, b$ such that $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ is an eigenvector of $A$.
A: $a = 3, b = 2$    B: $a = 1, b = 0$    C: Impossible    D: $a = -1, b = 0$    E: $a = 8, b = 7$

Problem 6: [2 points] There are two main baseball teams in New York. The teams are called the Yankees and the Mets. Baseball fans in New York always support one team or the other, but never both. Surveys revealed that in year 2000, 75% of the fans supported the Yankees while 25% supported the Mets. However, we also know that every year, 10% of the the Mets fans switch to supporting the Yankees. Also, every year, 5% of the the Yankees fans switch to supporting the Mets. Let’s encode the population preferences in year $n$ as the vector $\vec{x}_n = \begin{pmatrix} Y_n \\ M_n \end{pmatrix}$ where $Y_n$ and $M_n$ are the proportions of the Yankees and the Mets fans respectively.

What proportion of the Yankees fans would we need to have in order that the proportion does not change from year to year? (We call this an equilibrium or steady state of the system.)
A: $Y_n = \frac{2}{3}$    B: $Y_n = \frac{1}{3}$    C: $Y_n = \frac{4}{5}$    D: $Y_n = \frac{1}{2}$    E: $Y_n = \frac{1}{4}$