Quiz 4 (102) Group

1. \underline{answer: TRUE}
   This is a part of the Invertible Matrix Theorem

2. see problem #2 for individual quiz (sec 102)

3. \underline{answer: 1}

   The Rank Thm ⇒
   \[
   \dim \text{Nul}(A) + \text{rank}(A) = \frac{2016}{\# \text{of columns of } A}
   \]
   \[
   \dim \text{Nul}(A^T) + \text{rank}(A^T) = \frac{2015}{\# \text{of columns of } A^T}
   \]

   Subtracting these equations and using given assumption that \text{rank}(A) = \text{rank}(A^T)
   we get
   \[
   \dim \text{Nul}(A) - \dim \text{Nul}(A^T) = 2016 - 2015 = 1
   \]
answer: (1), (2), (3) and (4)  
(all equivalent to A being invertible)

see problem #7 for individual quiz  
(sec 102)

claim (4) says: there exists an invertible matrix B such that AB is invertible.

Then  
\[(AB)B^{-1} = A\]  
is also invertible.

see problem #8 for individual quiz  
(sec 102)

answer: 0

If you use expansion across 1st row, the determinant is
\[(-1)^{1+1} A_{11} + (-1)^{2015+1} A_{1,2015} = A_{11} - A_{1,2015} = 0\]
b/c cofactors (111) and (1,2015) are identical.

Alternatively: All the rows from 2nd to 2015th are proportional...