(1) false. take $\mathbf{v}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$, 
\[ \mathbf{v}_2 = \begin{pmatrix} 2 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad \mathbf{v}_3 = \begin{pmatrix} 3 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad \mathbf{v}_4 = \begin{pmatrix} 4 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}. \]

(2) true. $T(c \mathbf{x}) = cT(\mathbf{x})$ (definition)

\[ T(0) = T(0, \mathbf{x}) = 0T(\mathbf{x}) = \mathbf{0}. \]

(3) none

all violate conditions of linearity

\[ T_1: \quad x_1 - 2 \quad \Rightarrow T_1(0,0) \neq (0,0,0) \]
\[ T_2: \quad x_1x_2 \quad (\text{nonlinear term}) \]
\[ T_3: \quad 1 \quad \Rightarrow T_3(0,0) \neq (0,0,0) \]

none of these terms can be represented by a matrix-vector multiplication.

(4) rotation clockwise by $\pi/2$:
reflection through line $x_2 = -x_1$:

$$
\begin{align*}
A_2 &= \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}, \\
T_2(e_1^2) &= (0, -1), \\
T_2(e_2^2) &= (-1, 0)
\end{align*}
$$

composite transformation:

$$
A = A_2 A_1 = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}
$$

5 one-to-one: no free variables. In a square matrix, this means no zero row, which means columns span the codomain,
which means columns are l.i.

only such matrix is

\[
\begin{pmatrix}
0 & -2015 & 0 \\
0 & 0 & 2016 \\
2017 & 0 & 0
\end{pmatrix}
\]

onto means no zero row, so a $5 \times 9$ matrix requires 5 pivot columns

see section (102), #7

(1), (2), (3)

(1) States: if

\[
\begin{align*}
A x_1 &= b \\
A x_2 &= b
\end{align*}
\]

then $x_1 = x_2$, where $A = (v_1, \ldots, v_n)$. Since

\[
A (x_1 - x_2) = 0,
\]
and $x_1$ must equal $x_2$,

$$Ax^2 = 0$$

has only the trivial solution

$\Rightarrow$ columns of $A$ l.i.

(2) states

$$Ax^2 = 0$$

has only the trivial solution

$\Rightarrow$

columns of $A$ l.i.

(3) states that if $\exists B$ such that

$$Ax_1 = b$$

has a unique solution, there can be no $x_h \neq 0$ such that

$$Ax_h = 0$$

or else $x_2 = x_1 + x_h$ would also be
a solution of $A \cdot x = b$. Therefore,

$A \cdot x = 0$ must only have trivial solution

$\Rightarrow$ l.i.

also, if $A \cdot x = b$ has a unique solution
for some $b$, $A$ then must have $n$ pivot columns (no free variables) and so its columns must be l.i.