Problem 1: [1 point] Let \( N = (\vec{v}_1 \quad \vec{v}_2) \) be a \( 3 \times 2 \) matrix with columns \( \vec{v}_1, \vec{v}_2 \). Under what condition, if any, can we be sure that the equation \( N\vec{x} = \vec{b} \) is consistent?

A: \( \vec{b} = \vec{v}_1 \times \vec{v}_2 \).  
B: \( \vec{b} \neq \vec{0} \).  
C: Always consistent.  
D: \( \vec{b} = \vec{v}_1 + 2\vec{v}_2 \).  
E: Never consistent.

Problem 2: [1 point] Starting with \( A = \begin{pmatrix} 3 & 2 & -1 & 3 \\ 4 & 3 & -1 & 4 \\ -2 & -3 & -1 & -2 \end{pmatrix} \), elementary row operations are used to reach reduced row echelon form. Which is correct?

A: \( \begin{pmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \)  
B: \( \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \)  
C: \( \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \)  
D: \( \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \)  
E: \( \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \)

Problem 3: [1 point] Let \( \vec{z} \) be any vector in \( \mathbb{R}^3 \). If we have a set \( V \) of unknown vectors from \( \mathbb{R}^3 \), how many vectors must be in \( V \) to guarantee that \( \vec{z} \) can be written as a linear combination of the vectors in \( V \)?

A: 6  
B: 4  
C: It is not possible to make such a guarantee.  
D: 3  
E: 2

Problem 4: [1 point] Suppose you have a system of two linear equations in two unknowns. You solve it by performing elementary row operations on the augmented matrix. Along the way, you plot the two linear equations that you have at each stage of row reduction in turn. Put the three graphs in order, from first to last (the last one is in reduced row echelon form).

A: 2, 3, 1  
B: 1, 2, 3  
C: 3, 2, 1  
D: 1, 3, 2  
E: 2, 1, 3

Problem 5: [1 point] We make lemonade in three kinds: regular, extra sweet and extra sour. Each lemonade is a blend of lemon juice and sugar with a fixed amount of water. The table shows how many cups of each ingredient are needed to make each kind of lemonade.

<table>
<thead>
<tr>
<th>lemon juice</th>
<th>sugar</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>regular</td>
</tr>
<tr>
<td>0.8</td>
<td>0.2</td>
<td>extra sour</td>
</tr>
<tr>
<td>0.3</td>
<td>0.7</td>
<td>extra sweet</td>
</tr>
</tbody>
</table>

Late one day a customer wants a special lemonade which is made up with 60% lemon juice and 40% sugar. We have run out of regular lemonade. Can we make up the special lemonade from extra sour and extra sweet? If so, in what proportions (sour:sweet) should they be mixed?

A: 2:1  
B: 3:2  
C: 3:1  
D: There are infinitely many ways to do this.  
E: It is not possible.