Problem 1: [1 point] Given a matrix \( A = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \), what is \( A^9 \)?

A: \( \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \)  
B: \( \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \)  
C: \( \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \)  
D: \( \begin{pmatrix} 9 & 0 \\ 0 & 9 \end{pmatrix} \)  
E: \( \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \)

Problem 2: [2 points] Suppose \( B = \begin{pmatrix} 1 \\ r \\ r \end{pmatrix} \) and you are told that \( B \begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} \). Find \( r \).

A: \( r \) can have any value  
B: \( r = 4 \)  
C: This is not possible  
D: \( r = 2 \)  
E: \( r = 0 \)

Problem 3: [1 point] Let \( A, B, C \) be invertible 2x2 matrices. What is the inverse of \( AB^{-1}C \)?

A: \( C^{-1}B^{-1}A^{-1} \)  
B: \( A^{-1}BC^{-1} \)  
C: \( C^{-1}BA^{-1} \)  
D: \( CB^{-1}A \)  
E: \( AB^{-1}C \)

There are two main soccer teams in Manchester. The teams are called United and City. Soccer fans in Manchester always support one team or the other, but never both. Surveys revealed that in year 2000, 60% of the fans supported United while 40% supported City. However, we also know that every year, 20% of the City fans switch to supporting United. Also, every year, 10% of the United fans switch to supporting City. Let’s encode the population preferences in year \( n \) as the vector \( \vec{x}_n = \begin{pmatrix} U_n \\ C_n \end{pmatrix} \) where \( U_n \) and \( C_n \) are the proportions of United and City fans respectively.

Problem 4: [2 points] What percentage of fans supported United in 2001?

A: 68%  
B: 62%  
C: 75%  
D: 60%  
E: 50%

Problem 5: [2 points] What was the percentage of fans that supported United in 1999?

A: 57%  
B: 55%  
C: 60%  
D: 50%  
E: 40%

Problem 6: [2 points] What proportion of United fans would we need to have in order that the proportion does not change from year to year? (We call this an equilibrium or steady state of the system.)

A: \( U_n = \frac{3}{5} \)  
B: \( U_n = \frac{4}{7} \)  
C: \( U_n = \frac{2}{3} \)  
D: \( U_n = \frac{3}{8} \)  
E: \( U_n = \frac{1}{4} \)