Math 221
Lab 1

In this document we’ll go over the basics of MATLAB. There are 6 questions you need to complete for this assignment, which are included in the text below, and repeated all together at the end of the document.

You must submit the document on Connect, and it must be in PDF format. No other format will be graded by the markers. This lab must be submitted on Connect by 5pm on Friday, September 25th. Please include all of your answers on a single document, that includes your name, student ID, course section, and instructor, clearly indicated at the top of the document.

Naming your PDF file: Your file should be named as follows: LastFirstLab1.pdf where Last and First are your last and first names respectively. For example, my file would be called CoombsDanielLab1.pdf.

What to expect from this lab.

After completing this lab, you should be able to:

• start up MATLAB on your lab computer and enter commands in the MATLAB Command Window.
• generate vectors and matrices by typing element by element or using the colon operator.
• perform component-wise operations on matrices.
• use MATLAB built in functions to perform simple matrix-vector operations.
• create a matrix, perhaps using the zeros and ones commands
• access and change parts of a matrix
• use the det command to calculate the determinant of a matrix.

Entering vectors and matrices

MATLAB is a program for manipulating matrices. An $n$ by $m$ matrix is a box of numbers with $n$ rows and $m$ columns. Vectors are a special case where there is only one row (a row vector) or only one column (a column vector). Try typing

$A = [1 \ 2 \ 3]$ 

in the command window and then press the enter key. MATLAB answers with

$A = 
\begin{bmatrix} 
1 & 2 & 3 
\end{bmatrix}$

What has happened here? MATLAB has assigned the 1 by 3 matrix (row vector) $[1 \ 2 \ 3]$ to the variable $A$. Actually what you saw on the screen probably didn’t look exactly like this, since by default, MATLAB double spaces all the output. To change this to single spacing type

format compact
in the command window and then hit enter. The workspace window on the top left shows all the variables that have been defined. There should be an entry for the matrix $A$ giving information about the size (1 by 3) and how much computer memory it uses.

When inputing a matrix $A$ to MATLAB, we define it row by row and use a semi-colon \; to separate the rows. For example, the matrix

$$
\begin{pmatrix}
1 & 2 & 3 \\
4 & 5 & 6
\end{pmatrix}
$$

is defined in MATLAB with the command

$$
A = \begin{bmatrix} 1 & 2 & 3; \\ 4 & 5 & 6 \end{bmatrix}
$$

Notice that the semi-colon is placed after the last entry of a row and so first entry of a new row should come after a semi-colon. Also note that the variable $A$ has been re-assigned from its previous value. You can also see that the size of the variable $A$ has changed in the workspace window.

The semi-colon operator \; has another useful purpose - it suppresses the output of MATLAB when placed at the end of the command. For example, typing

$$
A = \begin{bmatrix} 0 & 0.25 & 0.5 & 0.75 & 1 \end{bmatrix};
$$

defines the vector $(0 \ 0.25 \ 0.5 \ 0.75 \ 1)$ in MATLAB, but silently. Try it yourself. This command is quite useful if you don’t care about the output, especially if it is very long. To check that $A$ has been defined correctly, type

$$
A
$$

MATLAB answers

$$
A = \begin{bmatrix} 0 & 0.2500 & 0.5000 & 0.7500 & 1.0000 \\
\end{bmatrix}
$$

confirming that the value of $A$ is again $(0 \ 0.25 \ 0.5 \ 0.75 \ 1)$. Note that MATLAB is case sensitive. Thus, if you type $a$ instead of $A$, MATLAB will try to access a different variable from $A$.

The command

```
clear
```

clears all the variable assignments. Try it, and note the change in the workspace window. It’s a good idea to type this before starting a new calculation, to make sure that there are no unexpectedly defined variables.

There is a shorthand notation for entering row vectors with evenly spaced entries. Try

$$
X = 1:10
$$

Get the idea? The colon operator \; is an important MATLAB operator useful to generate row vectors. It occurs in different forms. Try

$$
X = 0:2:20
$$

The number in the middle — in this case 2 — is the increment. It can be any size, even negative.

If $A$ and $B$ are two matrices with the same number of rows, then $[A \ B]$ is the matrix obtained by placing them side by side. The same thing works for any number of matrices. In fact the notation $[1 \ 2 \ 3]$ is a special case of this. MATLAB regards numbers as 1 by 1 matrices, and the matrix $[1 \ 2 \ 3]$ is obtained by placing three of these 1 by 1 matrices side by side.
Question 1

(a) Using the shorthand colon (:) notation, write the MATLAB code that assigns the matrix
\[
\begin{pmatrix}
12 & 8.5 & 5 & 1.5 & -2 & -5.5
\end{pmatrix}
\]
to the variable \( X \).

(b) Write the MATLAB code that assigns the matrix
\[
\begin{pmatrix}
1 & 6 & -2 \\
-1 & 10 & 0 \\
4 & 1 & -7
\end{pmatrix}
\]
to the variable \( Y \).

(c) Let \( A \) be the matrix produced by the MATLAB code
\[
A = [1:-1:-1; 3:3:9; 20:-20:-20];
\]
Write down the value of the element in the third row and second column of the matrix \( A \).

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Component-wise operations on matrices

In the course, we have introduced addition and scalar multiplication of matrices. Recall that these are component-wise operations. This means that if we add two matrices \( X \) and \( Y \) we simply add each component of \( X \) to the corresponding component of \( Y \). Notice that in order to add two matrices, their dimensions must be identical. Define \( X \) to be \([1 2 3; 4 5 6]\) and \( Y \) to be \([1 1 1; 1 1 1]\). Then add \( X \) and \( Y \) by typing \( X+Y \)

Subtraction works exactly the way you would expect it to, for example the command \( X-Y \)

subtracts \( Y \) from \( X \). Notice that if the result of a MATLAB calculation is not explicitly assigned to a variable, it gets assigned to the variable \texttt{ans} by default.

Scalar multiplication is also a component-wise operation. According to the definition, to multiply the scalar \( s \) times a vector (or matrix) \( X \) we must multiply each component of \( X \) by \( s \). Try \( 2*X \)

Check that \( X+X \) gives the same answer.

In MATLAB, if you have a vector or matrix \( A \), then MATLAB understands that adding a scalar to \( A \) means that you would like to add that scalar to every entry of \( A \). So, for instance, \([1 3] + 2 \) returns the matrix (vector) \([3 5]\).
Question 2

(a) If \( A = \begin{bmatrix} 1 & 2 & 5 \end{bmatrix} \) and \( B = \begin{bmatrix} 2 & 2 & -1 \end{bmatrix} \) (row vectors), write the MATLAB code that produces the output corresponding to

\[
\frac{1}{6} \left( -2A + \frac{1}{3}B \right)
\]

(b) Write the MATLAB code that produces the output of the matrix

\[
(6C - 3D)/5
\]

where

\[
C = \begin{pmatrix}
-1 & 0 & 1 \\
0 & -1 & 0 \\
1 & -1 & 1
\end{pmatrix}, \quad D = \begin{pmatrix}
0 & 1 & 0 \\
-2 & 3 & 2 \\
0 & 1 & -2
\end{pmatrix}
\]

Changing part of a matrix

Suppose \( A \) is a MATLAB matrix. We say a matrix is \( n \times m \) if it has \( n \) rows and \( m \) columns. Consider the following \( 3 \times 4 \) matrix \( A \) as an example

\[
\begin{align*}
\text{>> } A &= \begin{bmatrix} 1 & 2 & 1 & 2; 3 & 4 & 3 & 4; 5 & 6 & 5 & 6 \end{bmatrix} \\
A &= \\
1 & 2 & 1 & 2 \\
3 & 4 & 3 & 4 \\
5 & 6 & 5 & 6
\end{align*}
\]

(\textbf{Note:} whenever we display \texttt{>> matlabCode}, we just mean that inside the Matlab terminal, you type \texttt{matlabCode}. e.g. to follow along with the above, you would simply type into the terminal \texttt{A = [1 2 1 2; 3 4 3 4; 5 6 5 6].}

How can we extract pieces of \( A \)? Individual entries can be accessed by specifying the row \( n \), counting down from the top, and column \( m \), counting from the left, in the notation \( A(n,m) \). For example

\[
\begin{align*}
\text{>> } A(2,3) \\
\text{ans} &= \\
3
\end{align*}
\]

We can change the entry to, say, 7 by entering the command

\[
\begin{align*}
\text{>> } A(2,3) &= 7 \\
A &= \\
1 & 2 & 1 & 2 \\
3 & 4 & 7 & 4 \\
5 & 6 & 5 & 6
\end{align*}
\]

Notice than when we adjust a component of \( A \) and omit the semi-colon at the end of the command, MATLAB displays the new version of \( A \).

To extract a sub-matrix, we must specify a range of rows and columns. For example, (assuming you have changed the \( A(2,3) \) entry to 7)
Now let’s set all these entries to zero.

```matlab
>> A(1:2,2:3)=[0 0; 0 0]
A =
    1 0 0 2
    3 0 0 4
    5 6 5 6
```

By the way, we could have used `zeros(2,2)` in place of `[0 0; 0 0]`, *i.e.* used the command

```matlab
>> A(1:2,2:3) = zeros(2,2)
```

The function `zeros(m,n)` is built into MATLAB which creates a $m \times n$ matrix with all zero entries. A similar function is `ones(m,n)`, except that `ones` creates a $m \times n$ matrix with all unit entries. So if we wanted to turn the entries of $A$ previously changed to zeros into ones, we could use the command

```matlab
>> A(1:2,2:3) = ones(2,2)
A =
    1 1 1 2
    3 1 1 4
    5 6 5 6
```

Remember you can get information about any function in MATLAB, like `ones`, by typing `help ones`.

When specifying ranges of rows and columns, we may use `end` to denote the last row or column. This is handy if we are not sure how long the matrix is. Also, we can mix the notations for single rows or columns, and ranges. For example, to extract the first row of $A$, that is, $A(1:1,1:4)$ we could equally well use $A(1,1:end)$. Even more succinctly, the range $1:end$ can simply be specified by `:`.

```matlab
>> A(1,:) = A(2,:) - A(1,:)
A =
    1 1 1 2
    2 0 0 2
    5 6 5 6
```

This a handy notation if you want to perform operations on a single row or column of a matrix. You may want to modify this matrix so that the second row is the result of subtracting the first row from the second. This is achieved by

```matlab
>> A(2,:) = A(2,:) - A(1,:)
A =
    1 1 1 2
    2 0 0 2
    5 6 5 6
```

Try it. You may have already learnt in the lectures, or you will soon, that performing operations on the individual rows of a matrix is a common technique to find the solution of a linear system.
Question 3

Let $B$ be the matrix

$$B = \begin{pmatrix} 1 & 1 & 0 & 1 \\ 2 & 4 & 3 & 4 \\ 5 & 0 & 1 & 1 \\ 2 & 1 & -1 & 1 \end{pmatrix}.$$ 

Denote the $i$-th row of $B$ by $R_i$ (i.e., $R_3 = (5, 0, 1, 1)$).

(a) Give the Matlab code that returns the second row of the matrix $B$.

(b) Write the Matlab code that replaces the second row $R_2$ of $B$ by the quantity $R_2 - 3R_4$, i.e., modify $B$ so that its second row is replaced with the second row minus 3 times the fourth row.

(c) With the modified matrix $B$ following part (b), write down the element of $B$ that is in the fourth column of the second row.

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Matrix-vector and matrix-matrix multiplication

The single asterisk $*$ denotes matrix-vector or matrix-matrix multiplication in MATLAB. The matrices must have the appropriate dimensions for this to take place, otherwise you will get an error message. As an example, try defining

$$M = \begin{pmatrix} 0 & 1 \\ 3 & 4 \end{pmatrix} \quad \text{and} \quad N = \begin{pmatrix} 1 & 0 \\ 2 & 2 \end{pmatrix}.$$ 

Now compute $M \cdot N$ and $N \cdot M$. Do you get the same answer? Should you get the same answer?

To multiply an appropriately-sized MATLAB column vector $\vec{x}$ by a matrix $M$, you similarly write $M \cdot \vec{x}$.

Check you can do this with MATLAB: if $\vec{x} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$, compute $MN\vec{x}$. You should get $\vec{x} = \begin{pmatrix} 0 \\ -3 \end{pmatrix}$. Can you verify that by hand as well?

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Dangerous Note: If you really want to do a component-wise multiplication of two matrices of the same size, MATLAB will comply, provided we use the operator $\cdot*$ (a dot followed by a single asterisk). Try

$X \cdot * X$

and check that each component of the result is the product of the corresponding components of $X$. Why is this dangerous? Because it is important that no-one confuses this component-wise multiplication with proper matrix multiplication! You are unlikely to need to do component-wise multiplication in this course.

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Question 4

In class we looked at a simple Leslie Matrix model for population dynamics of a species of bird. This question examines a similar model for an insect species. The population of insects has three classes to consider: eggs, nymphs and adults. Let these populations be given by $x_n, y_n$ and $z_n$ in month $n$. So the whole population structure can be written as a vector $\vec{x}_n = \begin{pmatrix} x_n \\ y_n \\ z_n \end{pmatrix}$. Suppose that each adult produces 1000 eggs per month.
Of the eggs, 2% survive each month to become nymphs in the next month. Then only 5% of the nymphs survive each month to become adults in the next month. 95% of adults die after producing eggs but 5% survive and remain as egg-producing adults in the next month. Suppose that at time zero, someone releases ten adult insects into a new habitat, that is, suppose the initial population is $\vec{x}_0 = \begin{pmatrix} 0 \\ 0 \\ 10 \end{pmatrix}$.

(a) Work out the Leslie matrix for this model and enter it into Matlab. In your solutions, give the MATLAB command that you entered.

(b) By repeatedly applying the Leslie matrix to $\vec{x}_0$, make a table showing the number of adult insects each month for the first months zero to 12.

(c) How many adults will there be after 20 years has passed? Hint: MATLAB will compute matrix powers input using the caret symbol, e.g. $A^2$, or you can compute by pencil and paper if you have a lot of time to spare.

Functions

MATLAB has a large number of built in functions. Let us explore just a few. Let’s begin with the dot product. If $X$ and $Y$ are vectors, then dot($X,Y$) computes the dot product of $X$ and $Y$. Try some simple examples. Does it matter whether $X$ or $Y$ are row or column vectors? Recall that a function can be thought of as a device that takes some input and uses it to compute some output. For the dot($\cdot,\cdot$) function, the input is a pair of vectors and the output is a number.

If you want to more information about a MATLAB function, you can use the built in help. If you know the name of a function, you can type

```matlab
help <functionname>
```

at the command prompt to get information about that function. For example, to find information about the function dot, type

```matlab
help dot
```

in the command window and information about how to use the function will appear.

Question 5

The common trigonometric functions $\sin$, $\cos$ and $\tan$ are available in MATLAB. The inverse functions are called $\text{asin}$, $\text{acos}$ and $\text{atan}$. Normally, these function take a real number as input and produce another real number as output. However, in MATLAB, if they are applied to a vector or a matrix, MATLAB computes the functions component-wise. For example, if $X= \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ then $\sin(X)=\begin{bmatrix} \sin(1) \\ \sin(2) \\ \sin(3) \end{bmatrix}=\begin{bmatrix} 0.8415 \\ 0.9093 \\ 0.1411 \end{bmatrix}$.

(a) If $A$ and $B$ are two vectors in $\mathbb{R}^n$, then the dot product between them is given in terms of their lengths ($\|A\|,\|B\|$) and the angle $\theta$ between them in space:

$$A \cdot B = \|A\| \|B\| \cos \theta .$$

\footnote{Note that we are ignoring male insects here – all adults in this model can produce eggs, and all surviving eggs grow up to become egg-producers. See \url{http://www.ramas.com/CMdem.htm} for some discussion of these and other issues to consider with Leslie matrix models.}
Using the functions \texttt{dot}, \texttt{acos}, and \texttt{norm}, write the MATLAB code that gives the angle between the vectors

\[
A = \begin{pmatrix} 1 \\ 3 \\ 1 \\ 4 \end{pmatrix}, \quad B = \begin{pmatrix} 1 \\ 0 \\ -2 \\ -2 \end{pmatrix}.
\]

(b) Let’s say we have angles \( \theta = (\theta_0, \theta_1, \theta_2, \theta_3) = (-0.5 \ 0 \ 0.5 \ 1) \), and that we want for each \( \theta_i \) to find the slope of the line that passes from the point \((0,0)\) through \((\cos \theta_i, \sin \theta_i)\), for each of \(i = 0, 1, 2, 3\). Give the MATLAB code that produces the vector \( S \) that consists of these slopes, i.e., the code \( S = \text{ something to do with } \theta \).

\begin{itemize}
  \item \textbf{det}
  
  The function \texttt{det(A)} returns the determinant of a square (same number of rows and of columns) matrix \( A \). You have seen in the lectures how to compute determinants of \( 2 \times 2 \) (2 rows and 2 columns) matrices. Later in the course you will see how to calculate determinants of larger, square matrices by hand but it can be done now using MATLAB.

\end{itemize}

\textbf{Question 6}

\begin{itemize}
  \item (a) Let \( I_n \) be the \( n \times n \) identity matrix, so that it has ones along the diagonal and zeros everywhere else. In Matlab, the command to create \( I_n \) is given by \texttt{eye(n)} (get it?). Give a formula for the determinant of \( I_n \) as a function of \( n \), e.g., \( \det(I_n) = \ldots \). You can find this by experimentation.

  \item (b) Is it true that \( \det(A + B) = \det(A) + \det(B) \)? To find out, generate some matrices at random and experiment. Briefly report on your experiments and state your opinion on this statement.

  \item (c) Is it true that \( \det(AB) = \det(A) \det(B) \)? To find out, generate some matrices at random and experiment. Briefly report on your experiments and state your opinion on this statement.

  \item (d) How is \( \det(A) \) related to \( \det(A^{-1}) \)? Here, \( A^{-1} \) is the inverse of \( A \) and can be computed in MATLAB using \texttt{inv(A)}. Again, generate some matrices at random and experiment. Briefly report on your experiments and say what you think the relationship is, and why.
\end{itemize}
Recap of MATLAB commands from this lab

Commands to create a row vector:

: The colon operator is used to create row vectors with evenly spaced entries. When no increment is specified, the components of the vector are spaced by a unit. So 1:5 generates the row vector 1 2 3 4 5. To obtain non-unit spacing, specify the increment in between colons. So 1:2:5 generates the row vector 1 3 5. Increments can be decimal or negative numbers. Colons can also be used in sequence inside brackets to create parts of a vector. So \( A = [1:0.5:2 \ 4:-1:2] \) generates the vector \( A = [1 \ 1.5 \ 2 \ 4 \ 3 \ 2] \).

Commands to create a column vector:

' When a vector (or a matrix) is followed by a single quote, the output is a vector (or a matrix) in which each original row has been switched to a column (also known as transpose).

Commands to create a matrix with more than one row:

; When a semi-colon is used inside brackets, it ends rows. So \( A = [1:0.5:2; \ 4:-1:2] \) generates the 2×3 matrix

\[
A = \\
1 \ 1.5 \ 2 \\
4 \ 3 \ 2
\]

zeros\((n,m)\) and ones\((n,m)\) can be used to create matrices with \( n \) rows and \( m \) columns with respectively all zero or all unit entries.

Operations on vectors and matrices:

+ matrix addition. \( A+B \) adds each component of \( A \) to the corresponding component of \( B \). \( A \) and \( B \) must have the same size, unless one is a scalar. A scalar can be added to any size matrix.

- matrix subtraction. \( A-B \) subtracts each component of \( B \) from the corresponding component of \( A \). \( A \) and \( B \) must have the same size, unless one is a scalar. A scalar can be subtracted from a matrix of any size.

* scalar multiplication. \( 3* A \) multiplies each component of \( A \) by 3.

det The command \( \text{det}(A) \) computes the determinant of the square (same number of rows and columns) matrix \( A \).

\( A(i,j) \) gives the entry of \( A \) in the \( i^{th} \) row and the \( j^{th} \) column.

\( A(i,:) \) gives the \( i^{th} \) row of \( A \).

\( A(:,j) \) gives the \( j^{th} \) column of \( A \).

\([n \ m] = \text{size}(A)\) returns the number of rows \( n \) and columns \( m \) of the matrix \( A \).

Built in functions:

\( \text{dot}(X,Y) \) dot product of vectors \( X \) and \( Y \). The output is a scalar.

\( \text{cross}(X,Y) \) cross product of vectors \( X \) and \( Y \). The output is a vector.

\( \text{sqrt}(X) \) computes the square root of \( X \). If \( X \) is a vector, the output is a vector whose components are the square roots of the components of \( X \).

Other useful commands:

; When a semi-colon is added at the end of a command line, it suppresses the output of the result.

\( \text{clear} \) clears the values of all variables.
Lab 1 Assignment Questions

You are required to submit a PDF. Please include all of your answers on a single document, that includes your name, student ID, course section, and instructor, clearly indicated at the top of the document.

When submitting, please be sure your document name ends in .pdf . The TA’s will only mark those documents that are submitted as a single file in this format. Your file should be named as follows: LastFirstLab1.pdf where Last and First are your last and first names respectively.

Q1 (a) Using the shorthand colon (:) notation, write the MATLAB code that assigns the matrix

\[ \begin{pmatrix} 12 & 8.5 & 5 & 1.5 & -2 & -5.5 \end{pmatrix} \]

to the variable X.

(b) Write the MATLAB code that assigns the matrix

\[ \begin{pmatrix} 1 & 6 & -2 \\ -1 & 10 & 0 \\ 4 & 1 & -7 \end{pmatrix} \]

to the variable Y.

(c) Let A be the matrix produced by the MATLAB code

\[ A = [1:-1:-1; 3:3:9; 20:-20:-20]; \]

Write down the value of the element in the third row and second column of the matrix A.

Q2 (a) If \( A = [1 \ 2 \ 5] \) and \( B = [2 \ 2 \ -1] \) (row vectors), write the MATLAB code that produces the output corresponding to

\[ \frac{1}{6} \left( -2A + \frac{1}{3}B \right) \]

(b) Write the MATLAB code that produces the output of the matrix

\[ (6C - 3D)/5 \]

where

\[ C = \begin{pmatrix} -1 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & -1 & 1 \end{pmatrix}, \quad D = \begin{pmatrix} 0 & 1 & 0 \\ -2 & 3 & 2 \\ 0 & 1 & -2 \end{pmatrix} \]

Q3 Let \( B \) be the matrix

\[ B = \begin{pmatrix} 1 & 1 & 0 & 1 \\ 2 & 4 & 3 & 4 \\ 5 & 0 & 1 & 1 \\ 2 & 1 & -1 & 1 \end{pmatrix}. \]

Denote the \( i \)-th row of \( B \) by \( R_i \) (i.e., \( R_3 = (5 \ 0 \ 1 \ 1). \)).

(a) Give the Matlab code that returns the second row of the matrix \( B \).
(b) Write the Matlab code that replaces the second row $R_2$ of $B$ by the quantity $R_2 - 3R_4$, i.e., modify $B$ so that its second row is replaced with the second row minus 3 times the fourth row.

(c) With the modified matrix $B$ following part (b), write down the element of $B$ that is in the fourth column of the second row.

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**Q4** In class we looked at a simple Leslie Matrix model for population dynamics of a species of bird. This question examines a similar model for an insect species. The population of insects has three classes to consider: eggs, nymphs and adults. Let these populations be given by $x_n$, $y_n$ and $z_n$ in month $n$. So the whole population structure can be written as a vector $\vec{x}_n = \begin{pmatrix} x_n \\ y_n \\ z_n \end{pmatrix}$. Suppose that each adult produces 1000 eggs per month. Of the eggs, 2% survive each month to become nymphs in the next month. Then only 5% of the nymphs survive each month to become adults in the next month. 95% of adults die after producing eggs but 5% survive and remain as egg-producing adults in the next month. Suppose that at time zero, someone releases ten adult insects into a new habitat, that is, suppose the initial population is $\vec{x}_0 = \begin{pmatrix} 0 \\ 0 \\ 10 \end{pmatrix}$.

(a) Work out the Leslie matrix for this model and enter it into Matlab. In your solutions, give the MATLAB command that you entered.

(b) By repeatedly applying the Leslie matrix to $\vec{x}_0$, make a table showing the number of adult insects each month for the first months zero to 12.

(c) How many adults will there be after 20 years has passed? Hint: MATLAB will compute matrix powers input using the caret symbol, e.g. $A^2$, or you can compute by pencil and paper if you have a lot of time to spare.

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**Q5** The common trigonometric functions $\sin$, $\cos$ and $\tan$ are available in MATLAB. The inverse functions are called $\text{asin}$, $\text{acos}$ and $\text{atan}$. Normally, these function take a real number as input and produce another real number as output. However, in MATLAB, if they are applied to a vector or a matrix, MATLAB computes the functions component-wise. For example, if $X = [1 \ 2 \ 3]$ then $\sin(X) = [\sin(1) \ \sin(2) \ \sin(3)] = [0.8415 \ 0.9093 \ 0.1411]$.

(a) If $A$ and $B$ are two vectors in $\mathbb{R}^n$, then the dot product between them is given in terms of their lengths ($\|A\|$, $\|B\|$) and the angle $\theta$ between them in space:

$$A \cdot B = \|A\| \|B\| \cos \theta.$$  

Using the functions $\text{dot}$, $\text{acos}$, and $\text{norm}$, write the MATLAB code that gives the angle between the vectors

$$A = \begin{pmatrix} 1 \\ 3 \\ 4 \end{pmatrix}, \quad B = \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}.$$  

(b) Let’s say we have angles $\theta = (\theta_0, \theta_1, \theta_2, \theta_3) = (-0.5 \ 0 \ 0.5 \ 1)$, and that we want for each $\theta_i$ to find the slope of the line that passes from the point $(0,0)$ through $(\cos \theta_i, \sin \theta_i)$, for each of $i = 0, 1, 2, 3$. Give the MATLAB code that produces the vector $S$ that consists of these slopes, i.e., the code $S = \text{something to do with } \theta$. 

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Q6 (a) Let $I_n$ be the $n \times n$ identity matrix, so that it has ones along the diagonal and zeros everywhere else. In Matlab, the command to create $I_n$ is given by `eye(n)` (get it?). Give a formula for the determinant of $I_n$ as a function of $n$, e.g., $\text{det}(I_n) = \ldots$. You can find this by experimentation.

(b) Is it true that $\text{det}(A + B) = \text{det}(A) + \text{det}(B)$? To find out, generate some matrices at random and experiment. Briefly report on your experiments and state your opinion on this statement.

(c) Is it true that $\text{det}(AB) = \text{det}(A) \text{det}(B)$? To find out, generate some matrices at random and experiment. Briefly report on your experiments and state your opinion on this statement.

(d) How is $\text{det}(A)$ related to $\text{det}(A^{-1})$? Here, $A^{-1}$ is the inverse of $A$ and can be computed in MATLAB using `inv(A)`. Again, generate some matrices at random and experiment. Briefly report on your experiments and say what you think the relationship is, and why.