Problem 1: Determine the Laplace transform \( F(s) = \int_0^\infty e^{-st} f(t)dt \) of the following by evaluating the integral:

1. \( f_1(t) = te^{3t} \)
2. \( f_2(t) = e^{-t} \sin(2t) + \cos(3t) \)

Problem 2: Use a Laplace transform table (available from the course home page where you found this homework, under Final Exam Resources), the linearity of the Laplace transform, and trigonometric identities to determine the Laplace transforms of these functions:

1. \( f_3(t) = 2t^2 e^{-t} - t + \cos(4t) \)
2. \( f_4(t) = e^{7t} \sin^2(t) \)
3. \( f_5(t) = \cos(mt) \sin(mt), \ m \neq n \)

Problem 3: Perform Laplace transform to calculate \( F(s) \) in the following case:

![Graph of f(t) for Problem 1](image)

Problem 4: Use a table of Laplace transforms to find \( y(t) \) in each case. Use partial fractions, frequency shift, time shift, or other rules as needed.

1. \( Y(s) = \frac{s}{(s+1)^2 + 1} \)
2. \( Y(s) = \frac{s^2}{(s+2)^2(s+4)} \)
3. \( Y(s) = \frac{3s+1}{(s-4)s^2+2} \)
4. \( Y(s) = \frac{1+e^{-2s}}{s^2-4} \)
5. \( Y(s) = \frac{1}{(s-3)^2} \)
6. \( Y(s) = \frac{s^2+2}{s^3} \)

Problem 5: An LRC circuit has inductance 0.5\( H \), resistance 1\( \Omega \), and capacitance 1\( F \).
(a) What is the transfer function $H(s)$?

(b) What is the impulse response $h(t)$?

(c) If the input voltage is $f(t) = \delta_3(t)$, what is the output $y(t)$? **Give an exact answer.**

(d) If the input voltage is $f(t) = e^{-t^2}$, what is the output $y(t)$? **Write your answer as a single integral – you do not need to calculate the integral.**

Problem 6 Calculate each convolution and evaluate:

(a) $t * e^t|_{t=4}$

(b) $\cos(t) * \sin(t)|_{t=3}$

(c) $t * t|_{t=2}$

Problem 7:

(a) Calculate the output $y(t)$ of a circuit with input $f(t) = \cos(\omega t)$ and transfer function,

$$H(s) = \frac{1}{s^2 + 2s + 2}$$

*Hint: use partial fractions to expand $Y(s) = \frac{A_s}{s^2 + 2s + 2} + \frac{B_3}{s^2 + 2s + 2} + \frac{C_3}{s^2 + \omega^2} + \frac{D_3}{s^2 + \omega^2}$.**

(b) What is the amplitude of $y(t)$ as $t \to \infty$?

(c) Plot your answer to part (b), and compare with the plot of $H(\omega)$. Is there a peak?