This week’s topics:

- Integrating factor method (linear 1st order)
- Slope fields
- Examples and applications

- Reading: Lebl 0.2, 1.1-1.4.

- Which of the following is an integrating factor for \( y' + 2y = 3t \)?
  A. \( e^{2t} \)
  B. \( e^{2t+5} \)
  C. \( e^2 e^{2t} \)
  D. \( 7 e^{2t} \)
  E. All of the above

Slope Fields (Lebl 1.2)

- 1st Order Equations
  
  \( (t, y) \) coordinate \( \rightarrow \) slope \( \frac{dy}{dt} \)

- Plot the slope as vectors – picture is the slope field
- Arrows always point right
- Initial data gives a starting point
- Can guess path of solution by following the arrows

Example

- Sketch the slope field of
  
  \[
  \frac{dy}{dx} = \frac{x^2}{y}
  \]

- Indicate the solution when \( y(0) = 1 \) and \(-1\).

  – Recall the general solution of the ODE from last time: 
  
  \[
  y = \pm \sqrt{\frac{2x^3}{3} + C}
  \]
Example

• Sketch the slope field of
  \[ \frac{dy}{dx} = \frac{x^2}{y} \]

• Indicate the solution when \( y(0) = 1 \) and -1.
  
  – The general solution of the ODE (separate variables):
  \[ y = \pm \sqrt{\frac{2x^3}{3} + C} \]

To do this in Matlab

• \( f = @(x,y) x^2/y; \) \hspace{1cm} \text{sets up function } f
• \text{dirfield}(f, -2:0.2:2, -2:0.2:2) \hspace{1cm} \text{uses dirfield.m}

• Notes:
  - -2:0.2:2 is a vector containing -2, -1.8, -1.6 ... 1.8, 2
\[ D = kV \]

**N2L:** \[ m \frac{dv}{dt} = mg - kV \]

\[ \frac{dv}{dt} = g - \frac{k}{m} v \]

Linear 1st order

If \( v < \frac{mg}{k} \), \( \frac{dv}{dt} > 0 \) and \( v \) increases.

If \( v > \frac{mg}{k} \), \( \frac{dv}{dt} < 0 \) and \( v \) decreases.

If \( v = \frac{mg}{k} \), \( v \) stays the same (terminal velocity).

Points where \( \frac{dv}{dt} = 0 \) are called steady states of the equation. So \( v = \frac{mg}{k} \) is a steady state. It is also a stable steady state. That means that small perturbations of the velocity, near \( v = \frac{mg}{k} \), will always be damped out and the velocity will return to \( \frac{mg}{k} \).

Solve the equation for \( v \). It's linear, so int. factor would work. It's also separable.
\[
\frac{dv}{dt} = g - \frac{k}{m} v \\
\int \frac{1}{g - \frac{k}{m} v} \frac{dv}{dt} dt = \int 1 dt \\
-\frac{m}{k} \ln(g - \frac{k}{m} v) = t + C \\
(g - \frac{k}{m} v) = e^{\frac{-k}{m} t + C} \\
V = \frac{mg}{k} - C_2 e^{\frac{-k}{m} t} \\
V = \frac{mg}{k} \left(1 - C_3 e^{\frac{-k}{m} t}\right) \\
\]

Observe: \[\lim_{t \to \infty} V = \frac{mg}{k}\] (terminal velocity)

If we know an initial velocity \(V(0) = V_0\),
we can solve for \(C_3\).
\[V_0 = \frac{mg}{k} \left(1 - C_3\right) \quad (t=0) \quad (v=v_0)\]
\[1 - C_3 = \frac{kV_0}{mg}\]
\[ C_3 = 1 - \frac{k v_0}{mg} \]

so

\[ v = \frac{mg}{k} \left( 1 - \frac{1 - \frac{k v_0}{mg}}{e^{\frac{k}{m} t}} \right) \]

If \( v_0 = 0 \):

\[ v(t) = \frac{mg}{k} \left( 1 - e^{\frac{-k}{m} t} \right) \]